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THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

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SEPTEMBER 1919

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VOLUME L SEPTEMBER 1919 NUMBER 2

ON THE CAUSE OF CEPHEID VARIATION

SECOND PAPER

By C. D. PERRINE

Several hypotheses have been proposed to account for the Cepheid type of variation, but as yet a decisive test has not been found and the cause remains uncertain. The discovery of the variations of the radial velocities of these stars and the possibility of representing these variations by orbital motion seemed to be a long step toward the solution of their peculiar light-variations.

The closeness with which these variations are represented by orbital motion, usually well within the known uncertainties of the velocities observed, is in itself, in the absence of proof to the contrary, almost conclusive evidence of their binary character.

Some investigators have found difficulties, however, in explaining some of the conditions encountered upon a binary assumption, which have led as far even as suggestions to overthrow entirely the evidence of variable radial velocities and to ascribe the observed shift of the lines to activity in a single body. So far as I am aware, however, it has not been proved conclusively that these stars cannot be binary.

When the early spectroscopic results showed such a startlingly large proportion of binaries, I must confess that I wondered for a time if any pulsation effect were possible in single stars which

could cause such shifts of lines. I finally came to the conclusion that the displacements of the lines must be due to orbital motion, because it was difficult to conceive of internal pulsations being as uniform both in length of period and in amount of variation as is the case in the Cepheid variables, and because the lines, even in the stars showing the greatest ranges of velocity, appear to show few signs of violent disturbance.

The irregularities in the spot period of our own sun and the certainty that the underlying action is a pulsation effect of some sort in a body whose spectral condition is in general closely allied to that of the Cepheid variables, as well as the irregular nature of the variations in brightness of the novae and of many of the variables of long period and late spectral type, lead me to think that pulsation effects in general would probably be much more irregular than is the case in the Cepheid stars. This, it seems to me, is what might be inferred from a consideration of the nature and causes of pulsation effects in general. There can be little doubt that orbital motion would produce much greater uniformity than purely internal action of any kind not due to the presence of a secondary body.

While it is true that there is not entire uniformity in the periods nor in the amount of variation of the Cepheids, the deviations are very small and can hardly affect the foregoing reasoning to any considerable extent.

It is to me inconceivable that pulsations in a single body sufficient to double the light could occur in the short periods of these stars, a few days or hours, with any such regularity as that observed.

I find but little published information as to the character of the lines in the spectra of the Cepheids. In most of the orbital investigations examined no mention whatever is made of the character of the lines. Wright remarks[1] upon the breadth and haziness of lines in the spectrum of η Aquilae, which appear to be more marked in some lines than others, and lays stress on the well-known difficulties due to blends. A careful investigation of the widths and structure of the lines in the spectra of these stars would be a useful piece of work.

[1] *Astrophysical Journal*, 9, 60, 1899.

If we consider the integrated effect of a general pulsation in a star's atmosphere, we must come to the conclusion, I think, that it would cause a considerable widening of the lines. Suppose, as the simplest case, that the pulsation is uniformly radial and outward. The light of such a star entering the slit of the spectrograph would be made up of a large amount from the central portion of the star's disk, which would give a displacement toward the violet, whereas the light from the edges of the disk would be nearly without Doppler-Fizeau effect and would, therefore, occupy a nearly normal position. As the variations of radial velocity in this class of stars are large, the effect of such an integration should be noticeable on the width of the lines.[1] There are also the pressure effects to be considered.

If the pulsations were uniform it might be conceivable that in some of these stars the lines could be wide enough to mask such pulsations. A consideration of the widenings of lines in the novae, the Wolf-Rayet stars and the bright-line hydrogen stars, stars which are undoubtedly very active, seems to render doubtful an assumption of uniformity of pulsations. Our sun is approximately of the same spectral type as the Cepheid variables, and there are certainly no pulsations in the sun to cause such changes of light. With activity sufficient to cause the general displacement of lines observed in the Cepheid stars, I should expect greater widening of the lines and more distortion generally than appears to have been observed.

However this may be, it is desirable definitely to confirm or negative the binary nature of these stars by independent evidence. To this end I have sought for other facts, and the object of this paper is to consider some such and other points relating to these stars.

IRREGULARITIES IN THE LIGHT-CURVES

Irregularities in the light-curves of such of these stars as have had orbits determined for them have been found, which seem to bear on the matter.

[1] Such an effect would lead to a preponderance of negative velocities from the Cepheids, which is not the case among those for which the velocities of the system are known.

The ratios of the masses of the components of all such stars are small, but there is an appreciable range in the values. If such irregularities in the light-curves agree with the relative masses which are indicated for the secondaries, this would be independent evidence of considerable weight that these bodies are in reality binary systems.

The irregularities which have been observed in the light-curves of some of these stars are in the nature of retardations after maximum or, in the most pronounced cases, an actual recovery of light, forming a "hump" in the curve.

In Table I is given a short summary of these characteristics for the stars in question. The stars are arranged in order of the ratios of masses $\frac{m_1^3 \sin^3 i}{(m+m_1)^2}$.

TABLE I*

Star	Mag. Max.	$\frac{m_1^3 \sin^3 i}{(m+m_1)^2}$	"Humps" in Light-Curves	Light Minima Rounder
α Ursae Minoris	2.1	0.00001	.....	.....
β Cephei	3.3	.0001	None	Yes
SU Cassiopeiae	5.9	.0003	None	Both rounder
SZ Tauri	7.2	.0004	None	.....
RR Lyrae	7.0	.0006	None	Yes, strong
Y Ophiuchi	5.8	.0011	Slight ?	Yes
X Sagittarii	4.0	.0016	None	Yes
RT Aurigae	5.0	.0018	Slight negative curvature	Yes
T Vulpeculae	5.5	.0018	One observer, yes; others, none	Yes
ζ Geminorum	3.7	.0023	None	Both rounder
δ Cephei	3.7	.0028	Yes	Yes
Y Sagittarii	5.8	.0040	Strong	Yes, strong
η Aquilae	3.5	.0043	Strong	Both much rounder
S Sagittae	5.6	.0049	Strong	.....
W Sagittae	4.8	.0050	Strong	Yes
SU Cygni	6.7	0.0058	Strong	.....

*The stars α Ursae Minoris and β Cephei appear to satisfy the essential characteristics of Cepheid variation and have been classed with these stars in the present investigations. For the purpose of charting, the eccentricity of β Cephei has been assumed as 0.05.

From these results it is seen that for all stars in which the mass-ratios are 0.0023 or less either there are no "humps" or they are so slight as to be doubtful, whereas those of 0.0028 and higher

all show markedly this characteristic. The evidence from so limited a number of stars could not well be numerically more definite in showing a relation of these humps in the light-curves to the relative masses of the secondaries of binary systems which have been determined from the radial velocities of these same stars on the assumption that the observed variations are due to a true Doppler-Fizeau effect. This is irrespective of whatever may be the way in which the secondary produces the fluctuations of light. As it should be on such an assumption, it is the larger secondaries which produce the humps.

The consistency of the evidence is such as to give confidence in its essential reality, notwithstanding the uncertainties of the orbital inclinations. It would be a strange coincidence that would cause such an agreement, if only fortuitous.

DIFFERENCES BETWEEN LIGHT-CURVES AND VELOCITY-CURVES

The general agreement of the light-curves and velocity-curves of the Cepheid variables has been frequently remarked by investigators. A lack of complete agreement in the times of maxima and minima has been observed also.

In addition to this lack of agreement of phase, I have found differences between the light-curves and velocity-curves of these stars which are probably significant.

These differences for the 13 stars for which I have orbits[1] may best be described as a tendency for the light-curve at minimum to be much broader and rounder than the velocity-curve at the same phase. This characteristic is universal in the 10 stars for which I have velocity-curves. In 8 out of these 10 the light-curves are sharper at maximum than the velocity-curves. These deviations are noted in the fifth column of Table I.

There appear also in several cases to be wider ranges of the individual light determinations at times of minima. A careful study of the accuracy of determinations and perhaps even more accurate observations will be necessary to establish the correctness of this suspicion. Some such condition is not impossible, however.

[1] *Astrophysical Journal*, **41**, 308, 1915.

ORBITAL ECCENTRICITIES

An examination of Campbell's Tables I, II, III, and V[1] shows two peculiarities, the first of which is especially striking:

A. The increase in eccentricities of the orbits of the stars in these tables with period, in the spectral classes B, A, and F, is not continuous but is abrupt, changing at periods of about 15 days. In each of Tables I, II, and III a line can be drawn such that all shorter periods have the eccentricities very small and all longer periods have eccentricities much larger. The eccentricities have been separated at the critical point and the results are given in Table II.

B. The amount of eccentricity appears to depend upon the relation of masses to separation, the systems with mass-ratios small in proportion to separations having larger eccentricities than those with more nearly equal masses for the components. This peculiarity needs careful study and is reserved for separate investigation.

After the foregoing discontinuity in the eccentricities was noticed it occurred to me that the eccentricities of the Cepheid variables were considerable on the average, that their periods were short, and that Campbell had omitted them from his Tables I, II, and III because of their peculiarities generally. They were then entered on the chart (Fig. 1) as circles. It is readily seen that they fill the gap satisfactorily. If a horizontal line is drawn at $e=0.16$ as far as periods of 17 days, *all of the Cepheids fall within the region of eccentricity 0.16 or greater and periods of 17 days or less*[2] *and only one star not known to vary in brightness falls well within the same area.* The conclusion is thus suggested that the variations of brightness and the larger eccentricities of the Cepheids separate them from other short-period binary systems which do not show variations of brightness, and that binaries with small eccentricities (except eclipsing stars) for some reason do not vary noticeably in light.

[1] *Lick Observatory Bulletin*, **6**, 36, 1910.

[2] The subsequent inclusion of Polaris and β Cephei as Cepheids modifies the literal correctness of this statement but not its essential significance.

TABLE II

Spectroscopic Binaries	Star	Vis. Mag.	Period	Eccentricity	
				Small	Large
Types O and B	β Canis Majoris	2.0	$0^d.25$	0.1±	
	u Herculis	Var.	2.05	.05	
	ψ Orionis	4.7	2.53	.06	
	γ Lacertae	4.7	2.62	.02	
	Algol	Var.	2.87	.04	
	α Virginis	1.2	4.01	.10	
	δ Orionis	2.5	5.73	.10	
	κ Cancri	5.1	6.39	.15	
	α Carinae	3.6	6.74	.18	
	η Orionis	3.4	7.99	.02	
	π_4 Orionis	3.8	9.52	.03	
	α Pavonis	2.1	11.75	.01	
	β Lyrae	Var.	12.91	.07	
	β Orionis	0.3	21.90		0.30
	B.D.−1°1004	5.0	27.16		.76
	ι Orionis	2.9	29.14		.75
	κ Velorum	2.6	116.65		.19
	φ Persei	4.2	126.5		.43
	ζ Tauri	3.0	138.		.18
	π Andromedae	4.4	143.72		.58
	μ Sagittarii	4.0	180.2		.44
	Mean			0.07	0.45
Type A	τ Tauri	4.3	1.50	0.08	
	δ Librae	Var.	2.33	.05	
	α_1 Geminorum	2.8	2.93	.01	
	β Aurigae	2.1	3.96	.00	
	ε Herculis	3.9	4.02	.07	
	α_2 Geminorum	2.0	9.22		0.50
	B.D.+66°664	4.9	11.6		large
	θ Aquilae	3.4	17.11		.69
	α Coronae Borealis	2.3	17.36		.33
	ζ_1 Ursae Majoris	2.4	20.54		.50
	β Ursae Majoris	2.4	27.16		.79
	α Draconis	3.6	51.38		.40
	η Virginis	4.0	71.9		.33
	α Andromedae	2.2	96.7		.51
	β Arietis	2.7	$107^d.0$		.88
	γ Geminorum	1.9	$3^y.5$		large
	Sirius	−1.6	$49^y.3$		.59
	Mean			0.04	0.55
Type F	13 Ceti	5.2	$2^d.08$	0.06	
	θ Draconis	4.1	3.07	.01	
	ζ_1 Lyrae	4.3	4.30	.00	
	ω Draconis	4.9	5.28	.01	
	ι Pegasi	4.0	10.21	.01	
	o Leonis	3.8	14.50	.02	
	χ Draconis	3.7	281.8		0.42
	δ Equulei	4.6	$5^y.7$		.36
	α Ursae Minoris	2.1	$11^y.9$		.35
	ε Hydrae	3.5	$15^y.7$		.6
	Mean			0.02	0.43

TABLE II—*Continued*

Spectroscopic Binaries	Star	Vis. Mag.	Period	Eccentricity	
				Small	Large
Types G to M	λ Andromedae	4.0	$20^{d}.54$	0.11	
	Capella	0.2	104.02	.02	
	β Herculis	2.8	410.58		0.55
	η Boötis	2.8	$489^{d}.14$		.18
	η Pegasi	3.1	$2^{y}.24$		.16
	β Capricorni	3.2	$3^{y}.77$		.44
	α Scorpii	1.2	$5^{y}.8$		.20
	ζ Herculis	3.0	34^{y}		.46
	Mean			0.06	0.33

It is of interest to examine the exception noted above (a_2 Geminorum), and another (B.D.+66°664), which, because the numerical value of the eccentricity was not given but simply stated to be "large," could not be properly charted. The periods

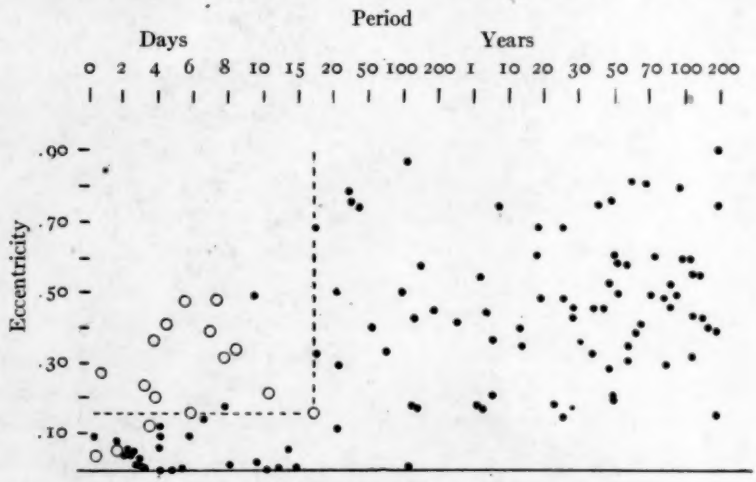

Fig. 1.—Orbital eccentricities of binary stars

○ = Cepheid variables

• = non-variables

of a_2 Geminorum, and B.D.+66°664, are 9.2 and 11.6 days respectively. It is thus seen that for periods of less than 9 days the separation at eccentricity 0.16 is essentially absolute for the data used.

As there are some variables which appear to belong to the Cepheid class with periods longer than 17 days, it does not seem likely that the lines of demarcation noted above are more than general, nor is it to be expected that it would be otherwise. Almost any physical hypothesis, especially if we conceive of a general unity of all these processes of change, would scarcely provide hard-and-fast limits for these phenomena. That Cepheid variation is practically limited to periods shorter than about 17 days is shown by Campbell,[1] who finds that out of 53 known Cepheids only 5 have periods longer than this.

The well-known limitations that the Cepheids, so far as data are available, have very small proper motions and that they are galactic in their preferences, furnish an explanation of these two exceptions. α_2 Geminorum has a galactic latitude no larger than some Cepheids, but its large proper motion, 0".203, differentiates it, probably in the matter of distance. It is also a multiple system. B.D.+66°664 has a small proper motion, 0".025, but its non-galactic tendency exempts it. It is to be noted that the spectral class of these two stars, class A, also tends to place them outside the region of this kind of variation, as practically all true Cepheids are of classes F, G, and early K.

The universally close relation between the velocity-curves and light-curves of all Cepheids spectroscopically observed justifies the conclusion, from their eccentric light-curves, that the orbital eccentricities of these stars are essentially all large.

These characteristics, especially that designated by B, almost undoubtedly have an important bearing on other and general problems related to evolution and to the physical condition at least of binary stars. A consideration of their bearing in these directions may well await advances in other investigations. Attention may be called, however, to the practically entire lack of small eccentricities among the stars of long period in the data above considered. The change in this respect at 17 days is as abrupt as in the complementary case of the non-variable stars of short period and small eccentricity. In fact it is so striking as to suggest the query whether there may not be some relationship—whether by some

[1] *Lick Observatory Bulletin*, **6**, 51, 1910.

process of selection or action the short-period stars have been converted from those of long period and large eccentricity. That the abrupt change of eccentricity with period is not a simple dependence upon length of period is abundantly clear. It seems

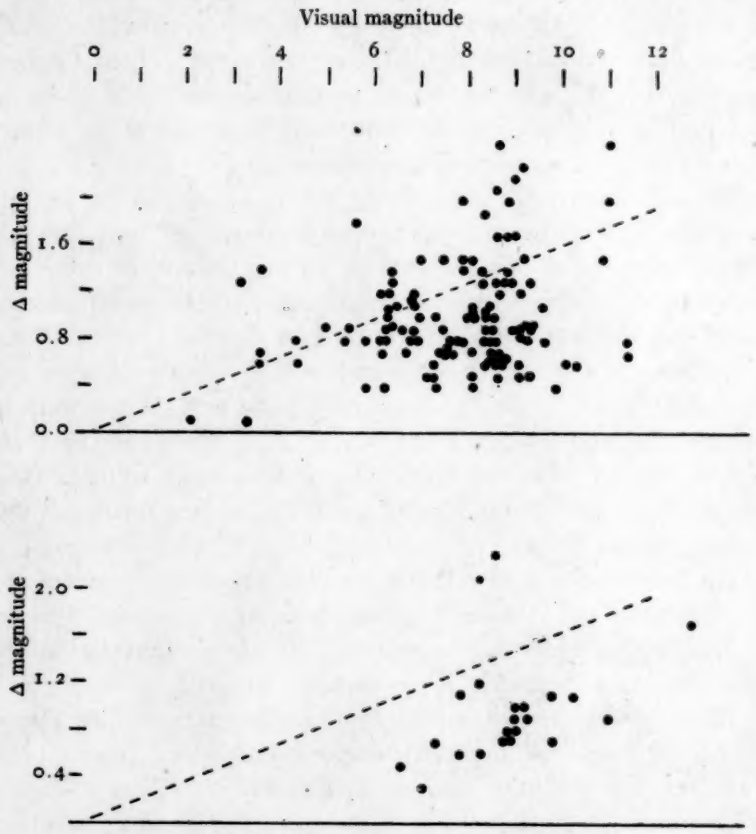

FIG. 2.—Variation of Δ magnitude with visual magnitude
Short-period variables: Galactic (upper figure)
Non-Galactic (lower figure)

to me equally certain that it is not a general physical condition such as might be expected to cause progressive change of spectrum or condition, but that it is probably some condition external to the stars.

The complementary relation of the Cepheids and non-variable stars of short period in the matter of orbital eccentricities is highly significant. As both spectra have been observed in short-period stars of small eccentricity, including Algol and β Lyrae variables, the binary interpretation of their line-shifts is placed beyond question. To assume that the line-shifts of Cepheid stars are not due to orbital motion is to assume that essentially all short-period binaries must have very small eccentricities as well as to reject entirely the interpretation of such consistent line-shifts.

In connection with the objection to duplicity of the Cepheid variables that their orbits appear to be almost impossibly small, the star β Cephei is of especial interest. Frost states[1] that "on some of the plates, certain lines have the appearance of complexity, and indicate the presence of a second-component spectrum." This star is of B type, all of which are under suspicion of being active. The very small size which is indicated for the secondary of this system makes it seem doubtful whether there is sufficient light from the secondary for its spectrum to be detected, unless through emission. This is not impossible, considering the type of spectrum of β Cephei and the fact that emission has been observed in such stars. Should Frost's observation be confirmed with regard to the spectrum of the secondary, we have valuable evidence of duplicity of stars of small orbital dimensions and secondaries, for the value of $a \sin i$ which he finds for β Cephei is only 45,000 km. The corresponding value of $\frac{m_1^3 \sin^3 i}{(m+m_1)^2}$ is 0.0001. These values are smaller than for any other Cepheid for which I find orbits, with the exception of the mass-ratio of Polaris. If this star is double, then small orbital dimensions certainly do not stand in the way of duplicity for the Cepheids. Frost discusses briefly the possible inclination of its orbit to the sight-line. A large inclination of orbit to the sight-line may be correct in the case of this star. Even so, as Frost concludes, its real orbit must be small. It cannot reasonably be urged that all the Cepheid orbits which have been determined have excessive inclinations to the sight-line. To do so would be to admit that these stars all

[1] *Astrophysical Journal*, **24**, 261, 1906.

have most improbable relations, not only to the observer, but to the plane of the Galaxy. Their orbits are undoubtedly small and the secondaries much smaller than the primaries.

β Cephei has been inserted in Fig. 3 as a circle. Its deviation from the other Cepheids in the way of a considerably larger orbital velocity for a small eccentricity, together with its very different spectral type, indicates not only a rather wide spectral range over which Cepheid variation can operate, but very close relations of the underlying cause and the cause of spectral condition generally.

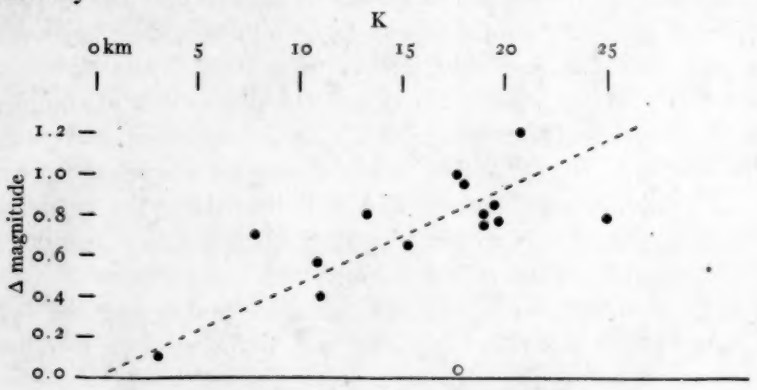

FIG. 3.—Variation of Δ magnitude with orbital velocity

The admission of β Cephei as a true Cepheid implies that this type of variation is largely dependent upon other than a narrow range of spectral condition. In my opinion this is in itself a strong argument against any theory of pulsation alone. A single star cannot be considered of much weight in a matter of the kind, but as far as it goes it is not inconsistent with greater activity and larger masses in the stars of spectrum of type B. It is also noticeable that the orbital velocities of the Cepheids appear not to differ materially from the other binary systems of the same spectral classes (F and G) and that as a rule the B and A stars have higher average orbital velocities than those of later type, indicating also larger masses. Here, with but few exceptions, we are again confronted by the unknown inclinations of the orbits of the binaries. That this uncertainty will to a large extent dis-

appear when we are dealing with considerable numbers of stars seems probable.

R Canis Majoris has a peculiar interest in this connection. It is an eclipsing star both by light-change and velocity-curve, is of Fo type, and the secondary has a mass-ratio of only 0.0027. It is of short period, 1.14 day, and has a small orbit, $a \sin i$ being only 443,000 km.[1] The $a \sin i$ and mass-ratio are about the average of the Cepheids. R Canis Majoris is shown not only by line-displacements but also by eclipsing phenomena to be binary. In other words its duplicity appears to be established in two entirely independent ways. If, then, it is binary there is nothing in small orbits to hinder the Cepheids from being binary also.

Perhaps one of the strongest bits of direct evidence in favor of duplicity of these stars is the fact that the ratios of the masses of the components come out so small; for it is only such stars which, upon the hypothesis of revolution in a resisting medium of some kind, can show a variation in brightness. If the surface areas are nearly equal, little or no variation is possible, for obvious reasons. It would appear to be even more definite in indicating that the brighter star is the larger and that the chief variations of light are in the larger bodies. From the point of view that the known spectroscopic binaries of classes F and G, with the necessary orbital conditions, have without exception been found to vary in brightness, this evidence becomes even stronger for such an explanation of Cepheid variability.

Probability.—On the theory of chance alone some of the Cepheid stars should be binaries. All of these stars investigated have shown variable radial velocities, as usually interpreted, and the opinion is general that all will show these same displacements when observed. Of the sixteen stars of this class whose orbits are available, the probability is strong that, on the theory of chance alone (unless the Cepheids have essentially no relation to other stars), one or more are in fact binary, irrespective of what interpretation may be placed upon the line-displacements. No physical reason has been adduced that I know of to show that none of these stars can be binary. If, therefore, there is no

[1] Jordan, *Publications of Allegheny Observatory*, 3, 49, 1912.

evidence that these stars cannot be binary, and a strong probability that one or more are in fact binary, there is no reason to assume that the line-displacements of all are not in fact due to orbital motion.

The difficulties of explaining some of the Cepheid peculiarities on a binary hypothesis must be positive and overwhelming before the rejection of all such evidence can be seriously considered.

In my opinion the following four pieces of evidence far outweigh any suspicions to the contrary and leave no room to doubt the binary character of the Cepheid variables:

1. Representation of the observed variable line-displacements satisfactorily by orbital motion.

2. The apparent relation between the relative masses of such secondary bodies and the irregularities in the light-curves.

3. The limitation of non-eclipsing variation in brightness among stars of short period to such as, upon binary assumption, yield large orbital eccentricities, whereas essentially all other short-period spectroscopic binary stars yield nearly circular orbits; and the abruptness of the limits.

4. Other evidence of a more general nature, such as the strong presumption of a similarity in constitution and evolutionary processes among all stars; the repeated observation of both spectra in short-period systems and in which the secondaries usually show much larger masses than in the case of the Cepheids, just as they should do upon such an explanation; spectral relations; orbital velocities, etc.

In the course of this investigation relations have been suspected between the amount of variability and brightness among the short-period variables and between the amount of light-variation and orbital velocities of the Cepheids which have had orbits determined.

Dependence of variability upon magnitude.—The greatest variations of brightness (Δ mag.) among the short-period variables appear to be among the fainter stars. The individual results taken from the list in the *Vierteljahrsschrift der Astronomischen Gesellschaft* for 1913 were plotted for the galactic and non-galactic stars separately, as in Fig. 2. The dependence does not appear

to be very close, as is evidenced by the considerable number of stars from sixth to ninth magnitude, with variations of brightness slightly below the mean. The stars farther from the galactic plane appear to show smaller variations of brightness on the average than do those nearer. Several possible explanations for such a relation suggest themselves. If the primary cause of variability is motion in a resisting medium, such a dependence may be due to a difference of size, the smaller bodies intercepting more matter in proportion to their masses than the larger ones, the activity being consequently greater in the smaller bodies. It might also result from the revolution of the primary of a binary system about the common center of gravity, the larger primary bodies having relatively smaller orbits and velocities than the small primaries. It is conceivable also that it might result from pulsation effects, the pulsation being greater in the smaller stars. The relation to orbital velocity makes it seem more probable, however, that the underlying cause is due chiefly to some external cause rather than wholly to an internal one.

RELATION OF VARIATION OF LIGHT TO ORBITAL VELOCITY

If the variation of light of these stars is largely due to rotation in a resisting medium, a relation between orbital velocity and amount of variation might be expected. The amount of material for such an investigation is limited to 16 stars. An inspection seems to indicate some such effect, as is shown in Table III, and in detail in Fig. 3.

TABLE III

No. of Stars	K	Δ Mag.	$\frac{m_2^3 \sin^3 i}{(m+m_2)^2}$
	km		
8.....	12.0	0.44	0.0010
8.....	20.4	0.80	0.0036

The present data (Fig. 3) indicate a fairly consistent linear relation between orbital velocity and brightness-variation with zero as origin for both factors. This indicates a simple dependence upon the speed with which a body moves through a resisting medium.

It is to be borne in mind that K is not the true orbital velocity except in cases where the orbital plane coincides with the line of sight, but is affected by the factor sin i. Also that there is a relation between velocity in the orbit and eccentricity. In several cases considerable changes (or divergences) have been observed in the light-curves. These do not, however, alter the apparent dependence upon orbital velocity.

Dependence of light-variation upon orbital eccentricity.—There is an apparent dependence of the brightness-variation upon orbital eccentricity, as shown in Fig. 4. If Polaris and β Cephei are

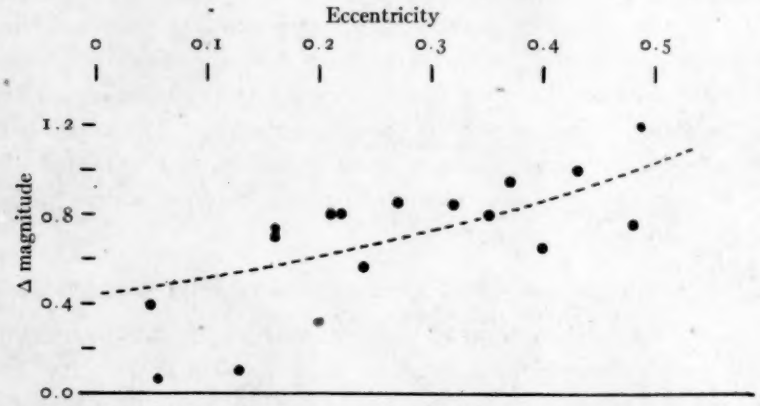

FIG. 4.—Variation of Δ magnitude with orbital eccentricity

omitted the remaining 14 stars show a curve which can be very well explained as a dependence upon eccentricity with the origin, for a circular orbit, at Δ magnitude of about 0.5. The dotted curve represents the variation of brightness with eccentricity on the assumption that it is due simply to the increase of velocity at periastron over that at apastron. The amount of data is too limited to determine the form of the curve, but seems to point fairly definitely to some sort of relation. Such a dependence upon orbital eccentricity is further evidence that the Cepheids are in reality binaries.

Dependence of light-variation upon period.—There appears to be no close relation between the amount of light-variation and length of period. The short-period variables with periods greater

than ten days show, however, a greater light-variation than do those of shorter period, but there does not seem to be any progression; it is apparently abrupt. This conclusion is based upon the 179 short-period variables given in the *Vierteljahrsschrift der Astronomischen Gesellschaft* for 1917. These may not all be true Cepheids, but it does not seem probable that the exceptions will materially affect the foregoing conclusion. The average magnitude-variation for the 179 stars is 1.04; 65 of these have periods of over 10 days, their average brightness-variation being 1.23 magnitude; 10 of these have nearly symmetrical light-curves and their light-variations are small. If these are rejected, the 55 remaining stars give an average brightness-variation of 1.33 magnitude. The 27 stars with periods of over 20 days (average 47 days) give essentially the same light-variation as those between 10 and 20 days. There are no indications of a dependence upon period among the stars with periods shorter than 10 days. Omitting the 65 stars with the longer periods, the remaining 114 stars with periods under 10 days give a mean light-variation of 0.93 magnitude.

The consistency of the larger light-ranges among the longer periods is shown by the fact that 44 of the 65 (or 55) have variations of 1.0 magnitude or greater.

The available data has been examined to determine, if possible, which conditions are the most important in causing this type of variation. As the Cepheids for which orbital data are available all show very small mass-ratios and orbital dimensions, these were taken as the basis, and stars of all spectral classes which have not shown variations in brightness, which come within these limits, were segregated. The upper limit for mass-ratio was set at 0.1, which is nearly twenty times that observed among the Cepheids. This was done with the view of testing the importance of this factor. The results indicate what was to be expected: that all of several conditions are necessary; that when all of these conditions are present the result is Cepheid variability, and, what seems especially significant, that ***no star was found, in which all of the conditions are present, whose light still remains constant.*** Several B and A stars were found with all the necessary orbital

and other conditions, but without known light-variation, except one of type B with a very small light-variation. On the other hand, not one F or G star with these conditions was found which did not vary in brightness. This seems to establish rather definitely that this limited spectral condition is a prime necessity to the production of full Cepheid variation. This examination further indicates that small proper motion is also an essential. Whether this means that their distances are great or that their motions are small, or both, cannot now be determined owing to the uncertainty which exists as to the relation of distance to proper motion.

The characteristics which stand out so strongly that they appear to be essential to the production of Cepheid variability are the following:

1. Short periods and small orbital dimensions.
2. Considerable orbital eccentricity.
3. Relatively small size of primary.
4. Preference for galactic regions.
5. Small proper motion.
6. Strong preference for F and G types of spectrum.
7. Small light-range.

Of these, after short periods, eccentricity and small mass-ratios seem to be the deciding factors, for we find the other conditions among the stars with periods of less than 10 days without their showing non-eclipsing variability. So far as the data goes the conclusion is very definitely indicated that very small secondaries are essential. Not a single case to the contrary exists among the 16 stars included in this investigation. As it is only the short-period stars with very small mass-ratios which also have large eccentricities, we are led to the conclusion that there is a close connection of some kind. The finding among stars of all periods, separations, and spectral types, of what appears to be a dependence of orbital eccentricity upon mass, mass-ratio, and separation seems to indicate that, in the Cepheids, the large eccentricities may be merely accidental and result from the limitation of this class of phenomenon to small secondaries. It is of much importance to decide this matter if possible, for upon it may depend to a large

extent a proper explanation of Cepheid variability. If large eccentricity is in reality the important factor, then some physical or tidal action either upon the primary or secondary is indicated, some internal cause, whereas if it is the size of secondary and not eccentricity, an external cause seems more probable. The galactic preference of these stars, as is pointed out elsewhere in this paper, favors the hypothesis of an external cause and, upon the foregoing reasoning, dependence upon relative size of secondary.

On account of the uncertainties of inclination in the spectrographic orbits it is difficult to obtain at present any very decisive results as to the law of this apparent relation of eccentricity to masses and separation. A casual examination shows this peculiarity in all spectral classes and among stars of all periods and masses. So far as a hasty examination of the data (the only one possible just now) shows, the phenomenon is entirely general in space distribution as well. If it is a fundamental law of nature, then in all probability the large eccentricities of the Cepheid orbits are entirely incidental and the controlling factor is size of secondary. A more careful study of the phenomenon and probably more data will be necessary for a definite answer. As has been stated already, this peculiarity is under investigation.

The almost complete restriction of the Cepheid type of variation to the intermediate spectral classes, F and G, when so many more stars of short period, at least among the spectroscopic binaries, are found in earlier types, particularly B, is in itself an interesting and important matter. Such a restriction can be accounted for most readily, it seems to me, upon some theory of activity among the early-type stars due to an external cause. Upon such an assumption the activity is likely to be much more general in the earlier types, particularly B, and to have affected all parts of their surfaces, whereas in the later and less disturbed types the action of a resisting medium might be expected to be in its earlier stages, due perhaps to a lesser density of the medium, and to affect chiefly the advancing face of the star.

In the Cepheids the explanation which seems most plausible on the whole is that proposed by R. H. Curtiss,[1] in which a resisting

[1] *Lick Observatory Bulletin*, 3, 40, 1904.

medium in the system has enhanced the advancing face of the primary. It is necessary to recognize that such a cause will affect the secondary as well and to a considerable extent, because of the rather large eccentricities of the orbits of these stars, that the resulting variation of brightness will be due to at least three factors, viz.: the brightening of the advancing face of the primary, a similar brightening of the secondary, and increased general activity in the secondary due to its higher orbital velocity and near approach to the primary at periastron. This hypothesis may be stated in the alternate form that Cepheid variation is the difference in brightness between the two components of unequal size of a binary system revolving in a resisting medium plus the increased activity of the secondary due to its higher orbital velocity, especially at periastron. If the variation in brightness of these stars were due chiefly to activity in the secondary after periastron passage as a result of the near approach to the primary, more cases of light-variation might be expected in the stars of classes B and A where the periods average even shorter than in the stars of type F and G, and where the orbital velocities are also much higher. This lack of full Cepheid variability among class B stars seems to be almost sufficient of itself to justify the conclusion that it is due to some form of surface activity on the advancing faces of these stars.

In my opinion an almost deciding factor as to the nature of Cepheid variation is that of their strong preference for the Milky Way. The mean galactic latitude of the 168 short-period variables listed by Harvard (in Vol. **56,** p. 191, of their *Annals*) is $16\frac{1}{2}°$. Eighty-eight of them are less than 10° from the plane. This fact is simple and direct and at once indicates that the cause is not wholly due to internal conditions or to the operation of general physical or gravitational laws, but that some external condition is the chief cause. In general the stars of spectral types F and G, to which the Cepheids belong by preference, are distributed nearly uniformly over the sky. That the Cepheids have a strong affinity for the Milky Way indicates that in some way their variability is influenced by conditions peculiar to that region. If it was due

simply to tidal or thermal action as a result of the near approach of the components at periastron, stars with the necessary orbital and other conditions anywhere in the sky should show this type of light-change. The nature of such an external condition is debatable. That it is matter in a form to act as a resisting medium seems most probable. Evidence is accumulating that such invisible matter exists in our stellar system in sufficient quantities to cause some of the phenomena peculiar to the Milky Way. In support of this is, first of all, the large number of meteors which the earth encounters daily. This is fairly positive and direct evidence of the existence of matter in a solid state and invisible. That such matter is confined to our solar system is not probable to say the least. Evidence is not lacking also as to the presence of matter in some sort of gaseous form but invisible. The detection of a considerable number of early-type stars whose spectra contain dark H and K lines of calcium, such lines showing either no radial velocity with respect to our stellar system or generally much smaller velocities than do the stars themselves, is evidence that the absorbing matter is essentially not connected with the stars. The small variations of radial velocity which are generally shown by these calcium lines in such stars and the fact that they usually exhibit the same general phases as for the star itself may conceivably be due to a motion of the calcium matter set up by the star. It is also significant that such calcium absorption has been observed in several of the novae of recent years. That the matter giving rise to this absorption is not connected with the earth's atmosphere or the solar system is obvious from the fact that all stars do not show it. As this characteristic has, so far as I am aware, been observed only among the early-type stars which have a strong affinity for the Milky Way, the inference is suggested that this may be another galactic peculiarity.

There is a growing belief that the dark lanes and "holes" of the Milky Way and the sharply defined dark patches in some of the large irregular nebulae are none other than masses of obscuring cosmical matter. This view is further strengthened by the large number of spiral nebulae which show dark lanes and rings. Just

what the action of a resisting medium would be in causing variations of brightness cannot be easily predicted. R. H. Curtiss[1] suggests simply an enhancing of the brightness of the advancing face of the primary. J. C. Duncan[2] favors a modification of that hypothesis. He considers it unlikely that the light-changes can be due to an actual change in the rate of emission since in that case the character of the spectrum should change radically during the interval between maximum and minimum, and favors a hypothesis based upon a variation of absorption in the star's atmosphere, brought about by a very rare envelope of nebulous matter brushing backward the star's atmosphere from its advancing face, thus reducing the amount of absorption on that face. It is probably too soon to venture an attempt to decide anything more regarding this type of variability than very general questions. There seems to me little doubt that the fundamental cause of these light-changes is to be found in the action of a resisting medium of some kind and that it may be considered to be sufficiently established that these stars are binary. The action assumed by Duncan seems to me less simple and direct than that of a brightening of the advancing face due to the addition of cosmical matter. Nor do I see a sufficient reason to doubt such an effect, especially if the matter encountered is solid and in small particles. If in this form, it seems to me doubtful if there would be sufficient "drag" to carry a gaseous envelope backward to the extent required. It seems probable that a very much smaller quantity of such matter would suffice for the observed increase of light if its entire kinetic energy were converted into radiant energy.

As to the change of spectral type, it is assured that in a number of these stars there is a slight change toward an "earlier" type at maximum. This change is certainly in the right direction. May it not be that it is sufficient also? It seems to me highly probable that this is all the change of spectrum required under an assumption of finely divided solid matter. That such matter is in a gaseous state seems doubtful from the fact that no emission

[1] *Lick Observatory Bulletin*, 3, 40, 1904.

[2] *Ibid.*, 5, 91, 1908.

lines have been detected in this class of star. Loud concludes,[1] I understand, that "only one of the pair is luminous," and "the visible star, then, is the satellite." This I think is contrary to the view now generally held. It would be difficult to explain why a small satellite should be so much brighter as to be the only one visible. This conclusion of Loud's appears to arise from his assumption that "according to the hypothesis to be tested, this component owes its light to the resistance of a diffused medium, to which the other must be assumed to be relatively at rest." It seems to me improbable that the "resisting medium" should be so related to such a system that the primary is relatively at rest with respect to it. If the "resisting medium" were of the nature of the zodiacal light appendage to our sun, then it might be, but there is considerable evidence that such a medium is in that sense disconnected and has an entirely separate existence. I think we may safely conclude that it is the primary which is not only the larger but the brighter, and that any tenable hypothesis must consider both as being luminous, even if one only is dominant. Duncan objects,[2] among other things, to the bombardment hypothesis on the ground that bombardment ought to render the spectrum of the companion visible. If the explanation offered in the earlier part of this paper of the cause of the "humps" in the light-curves of these stars is correct, we have positive evidence on that point. In that case the principal light-variations are due to the brighter body, and the fainter components offer only the small changes represented by the humps. These would undoubtedly be so small relatively as not to affect the spectrum appreciably.

In my opinion some such variations in the amplitude and character of the light-curves as have been observed in some of these stars are to be expected from the hypothesis of ~~rotation~~ revolution in a resisting medium and do not at all militate against their duplicity. It is not to be supposed that almost any such medium of which we can conceive would be perfectly homogeneous. If not homogeneous, variations due to this cause alone must occur in the brightness of a star moving in such a medium.

[1] *Astrophysical Journal*, **26**, 371, 1907. [2] *Lick Observatory Bulletin*, **5**, 91.

It is scarcely necessary to point out the need for more spectrographic orbits of Cepheid stars, even if they were only approximate. This is particularly true of such of these stars as have nearly symmetrical light-curves (small eccentricities) and those with the most unsymmetrical curves (largest eccentricities). Unfortunately most of these stars are faint. There are, however, a number of them which are within reach of the largest telescopes and one-prism dispersion. It would also be of considerable interest to determine orbits of a few stars from velocities derived separately from lines as high in the spectrum as possible and as low as possible, with a view to determining whether there is any difference either in the form of the orbit or the phase, as has been found in the observations of brightness for some of these stars. Refined observations of brightness of any or all of the stars whose spectrographic orbits have been determined would assist greatly in clearing up the questions which have arisen in regard to the cause of Cepheid variability.

CONCLUSIONS

1. Among the Cepheids for which orbits have been determined, those which have the largest relative masses for the secondaries show irregularities in their light-curves.

2. Among 16 Cepheid variables for which orbits are available, only two have eccentricities smaller than 0.13. The mean eccentricity of the 14 stars is 0.30. The eccentric nature of the light-curves of Cepheids confirms considerable eccentricities as a characteristic of these stars as a class.

3. Among 41 stars with periods of less than 17 days whose orbits are available, with but one exception all whose light is sensibly constant have small eccentricities.

4. Among 79 stars with periods from 17 days to 200 years, only three have eccentricities smaller than 0.15, and only one of these is smaller than 0.1. There appears to be among these stars a sharply defined lower limit of eccentricity at about 0.15. This limit is well marked among all periods longer than 15 days.

5. There appears to be among binary stars of all conditions a dependence of the orbital eccentricities upon the masses of the components and their separations.

6. There appears to be in the short-period variables a dependence of the amount of variability upon brightness, the fainter stars showing the greatest variation.

7. Among the Cepheids whose orbits are available, there appears to be a relation between the orbital velocity and the amount of variation, the stars whose orbital velocities are greatest showing the greatest variations.

8. There appears to be a tendency among the short-period variables for those with periods longer than about 10 days to have slightly larger light-variations. The effect appears not to be progressive with length of period, but to be more or less abrupt.

9. There are some indications of a dependence of light-change upon orbital eccentricity among the 16 Cepheids whose spectrographic orbits are available.

10. Evidence which is not entirely dependent upon the construction placed upon the periodic line-displacements in the spectra of the Cepheids confirms the binary nature of these stars.

The spectroscopic orbit of SU Cassiopeiae by Adams and Shapley[1] was received when this investigation was nearly completed. This star also falls into line in respect to relation of humps to size of secondary. Parkhurst's curve[2] shows no irregularities greater than can be ascribed to accidental error. The light-range is small, 0.47 magnitude according to Parkhurst and 0.33 magnitude by Müller and Kempf.[3]

These results are of especial interest on account of the small light-range, the low orbital velocity, and small eccentricity. The small light-range agrees with the other results as to a dependence of light-variation upon orbital velocity.

Small eccentricity in even a single one of these stars is of particular interest as it tends to confirm the explanation of motion

[1] *Astrophysical Journal*, **47**, 46, 1918. [2] *Ibid.*, **28**, 279, 1908.

[3] *Astronomische Nachrichten*, **173**, 307, 1907.

in a resisting medium, for if the variation were entirely dependent upon increased physical action due to a much closer approach at periastron, a nearly circular orbit should yield no sensible variation of light. If, however, the cause is due to orbital motion in a resisting medium, circular orbits might be expected to yield almost as large variations of brightness as highly eccentric orbits.

Observatorio Nacional Argentino, Córdoba
October 25, 1918

STUDIES BASED ON THE COLORS AND MAGNITUDES IN STELLAR CLUSTERS[1]

FOURTEENTH PAPER: FURTHER REMARKS ON THE STRUCTURE OF THE GALACTIC SYSTEM

By HARLOW SHAPLEY and MARTHA B. SHAPLEY

The present paper contains miscellaneous notes relating to or extending the results and arguments of the twelfth paper of this series. To economize space the observations are treated synoptically, and the discussion of interpretations is abbreviated in so far as consistent with a fair presentation. The subjects discussed are the following:

I. The indication of a direct relationship between globular and open clusters afforded by the galactic distribution.

II. The seventeen clusters which recent studies have added to the original lists in *Mount Wilson Contribution* No. 152.

III. Diagrams of the positions in space of all globular clusters.

IV. Evidence that the observed absence of globular clusters from the equatorial segment is not due to obstructing cosmic clouds in the Milky Way.

V. The diameter-parallax relation for globular clusters as derived from other sources than the Franklin-Adams charts.

VI. Holetschek's magnitudes of 40 globular clusters and the possibility of determining relative parallaxes from measures of the integrated light.

VII. Observational data supporting the view that the motions and distribution of spiral nebulae depend upon dynamical causes quite different from those prevailing in clusters of stars.

VIII. The bearing of the cluster studies on the formulation of a tentative hypothesis of the origin and development of the galactic system.

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 161.

IX. Evidence from the *Henry Draper Catalogue* concerning the extent and inclination of the local cluster defined in *Mount Wilson Contributions*, No. 157.

I. THE COMPLEMENTARY DISTRIBUTION IN GALACTIC LATITUDE OF GLOBULAR AND OPEN CLUSTERS

Although we have not as yet determined individual distances for open clusters we may safely accept that, generally speaking, they are much nearer the sun than the globular systems. If we should assume that the two kinds are similarly distributed with respect to the galactic plane,[1] most open groups would appear in relatively high galactic latitude, because of their proximity to the sun. With the exception of such near and luminous groups as the Pleiades and Praesepe, however, they occur only in low galactic latitudes, and they do not show the avoidance of the Milky Way exhibited by globular clusters. In fact, when actual positions in space are considered, they appear to occur in the dense stellar regions where globular clusters are not found; and they do not occur in the extra-galactic realms where the globular clusters are. This apparently complementary distribution of the two types of clusters is of high importance for the hypothesis which derives the galactic system originally from stellar clusters and obtains the open groups of the Milky Way from the extra-galactic globular organizations.

The continuous gradation of the compact groups into the open clusters of the Milky Way might be more clearly shown if we could correlate compactness with $R \sin \beta$;[2] but lacking parallaxes of the open clusters, we must for the present content ourselves with pointing out the complementary character of the frequencies of galactic latitude. In Fig. 1 the full line refers to open clusters—the objects of Classes II and III of Melotte's catalogue.[3] A few clusters have been withdrawn from his classes and two or three added, on the

[1] Cf. *Contributions from the Mount Wilson Solar Observatory*, No. 152, Figs. 4, 5, and 6; No. 157, Fig. 2.

[2] R is the radial distance from the earth, β is the galactic latitude.

[3] *Memoirs of the Royal Astronomical Society*, **60**, Part V, 1915. Diagrams by Melotte illustrate the well-known difference for open and globular clusters of the distribution in galactic longitude.

basis of Mount Wilson plates. Table I contains the assembled results.

The dotted line in Fig. 1 refers to globular clusters; the data are from *Mount Wilson Contributions*, No. 152,[1] supplemented by the additional material in Table II of this paper, omitting only the five

TABLE I

FREQUENCY IN GALACTIC LATITUDES OF OPEN AND GLOBULAR CLUSTERS

GALACTIC LATITUDE	NUMBER OF OPEN CLUSTERS		NUMBER OF GLOBULAR CLUSTERS	
	North	South	North	South
0°– 2°	21	25	0	0
2 – 4	19	19	1	1
4 – 6	10	13	4	2
6 – 8	7	4	3	5
8 –10	3	7	4	5
10 –12	5	0	4	3
12 –14	4	2	2	3
14 –16	0	3	4	2
16 –18	3	3	2	3
18 –20	0	0	1	1
20 –22	2	0	0	3
22 –24	0	1	2	0
24 –26	0	0	1	1
26 –28	1	0	0	1
28 –30	0	0	0	3
30 –32	0	0	1	0
32 –34	2	0	0	0
34 –36	0	0	1	2
36 –38	0	0	1	1
38 –40	0	0	1	0
40 –42	0	0	1	0
42 –44	0	0	0	0
44 –46	0	0	0	1
46 –48	0	0	1	1
48 –50	0	0	1	1

unproved clusters. Fig. 2 gives the frequency when the results are treated irrespective of the sign of β, and shows more clearly the apparent relation of open and globular clusters. A few globular systems in high galactic latitude are not represented in the diagrams, and for $\beta \geqq 10°$ the numbers of Table I for both globular and open clusters have been combined into small groups in drawing the curves.

[1] A few of the latitudes of Table V (*op. cit.*) have been slightly revised, as noted in the third section of the present contribution.

If for β, the angular distance from the plane, we were able to substitute the linear distance, $R \sin \beta$, the mid-galactic maximum for the open clusters would be closely restricted around the vertical axis; and the two maxima of the dotted curve would be relatively much steeper on the side nearer the vertical axis, showing that the galactic zone is entirely clear of globular clusters.[1]

Clusters of Melotte's Class II are more compact and populous than those of Class III, but the distribution in galactic latitude is

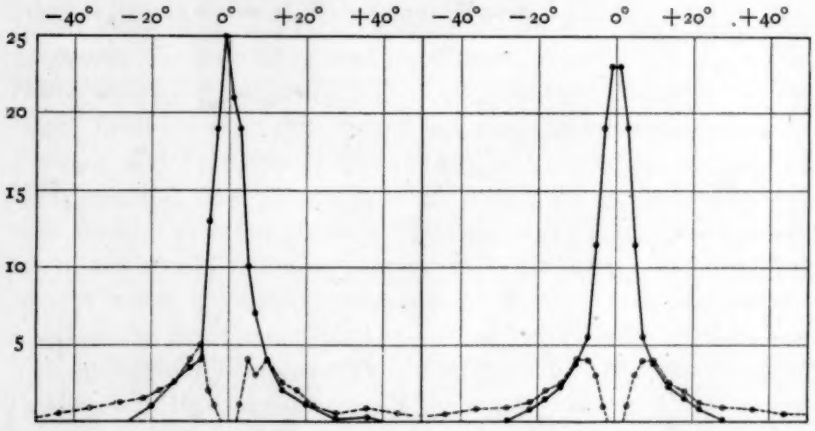

FIG. 1.—Frequency in galactic latitude of open clusters (full line) and globular clusters (broken line). Ordinates are numbers of clusters; abscissae are angular distances from the galactic plane.

FIG. 2.—Reflected frequency-curve of galactic latitudes for open clusters (full line) and globular clusters (broken line).

essentially the same for both types. Choosing from all the data the twenty richest and most regular open clusters, we find the following distribution in galactic latitude:

β	$<-4°$	$-4°,-3°$	$-2°,-1°$	$-0°,+0°$	$+1°,+2°$	$+3°,+4°$	$>+4°$
Number of Clusters	2	4	4	3	5	2	0

This certainly does not show a concentration to the medial plane; but perhaps it does not indicate as marked an avoidance of that plane as might be expected if we assume that globular systems are more closely allied to rich clusters than to the average open group.

[1] Cf. Fig. 6 of the seventh paper of this series.

II. THE DISTANCES OF 17 ADDITIONAL GLOBULAR CLUSTERS

A year ago 69 clusters were accepted as definitely globular.[1] Since that time special efforts have been made to examine with the 60-inch reflector the suspected and doubtful cases north of declination $-30°$ that had not been admitted to the earlier lists. The experience gained as a consequence of photographing faint and abnormal systems has also afforded a basis for better judgment in accepting or rejecting the clusters too far south for observation at Mount Wilson. Some unpublished observations of clusters with the Crossley reflector at the Lick Observatory were generously placed at our disposal by Professor Curtis. While at Mount Wilson during the past summer Professor Duncan, of Wellesley College, kindly made some long exposures on clusters with the 60-inch reflector.

Results of the survey of doubtful cases are summarized in Table II. In addition a number of clusters concerning which some doubt existed,[2] such as N.G.C. 4147, 6144, 6656, 6712, and 7006, have been proved during the year to be entitled to places in the lists; none of the 69 original clusters should be withdrawn, as far as the present observations go.

As in *Mount Wilson Contributions*, No. 152, the treatment of individual cases must be omitted. The distances, which in general cannot claim as high accuracy as those previously studied, depend for the most part upon diameters, since the objects in many cases are so faint that accurate analysis of the magnitudes would have been too laborious. Exposures of several hours were occasionally necessary to test for globularity. Frequently systems that shorter exposures had shown with some definiteness to be open[3] proved upon more persistent observation to be globular. At present there is hardly another object, north of $-30°$ at least, that is suspected with good reason of being a globular cluster.

Three points relative to the large distances are worthy of comment. (1) Practically all of these additional systems are more distant than 30,000 parsecs (100,000 light-years), thereby confirming an

[1] Tables V and VIII of *Mt. Wilson Contr.* No. 152, 1917.

[2] *Op cit.*, p. 9, n. 2.

[3] *Op. cit.*, p. 14, n. 2.

earlier surmise as to the completeness of the survey of globular clusters within that distance of the sun. (2) Every recognized globular cluster except one bears a number from the *New General Catalogue*, thus testifying that, in spite of distance and faintness, all of these remote objects were known prior to 1888. In fact, all but this one exception were known before 1864, the date of the *General Catalogue*, and all but three or four had already been catalogued by the

TABLE II

PARALLAXES OF 17 CLUSTERS

N.G.C.	R. A. 1900	Decl. 1900	Galactic		Distance (Unit is 100 Parsecs)		
			Long.	Lat.	Radial	Projected on Galactic Plane	From Galactic Plane
*5466	14h 1m.0	+29° 0′	10°	+72°	195:	60	+180
†I.C. 4499	14 45.0	−81 49	274	−20	250	235	− 85
*5927	15 20.8	−50 19	294	+ 5	185	185	+ 16
‡5946	15 28.2	−50 19	295	+ 4.0	415	415	+ 29
‡6355	17 17.8	−26 15	327	+ 4.4	500	500	+ 38
*6366	17 22.4	− 4 59	346	+16	320	310	+ 89
*6426	17 39.9	+ 3 13	356	+15	570	550	+150
*6440	17 42.9	−20 19	336	+ 2.5	525	525	+ 23
‡6496	17 51.8	−44 14	315	+11	305:	300	+ 58
*6517	17 56.4	− 8 57	347	+ 5	625	625	+ 55
‡6535	17 58.7	− 0 18	354	+10	370:	365	+ 64
†6539	17 59.4	− 7 35	348	+ 6	400	400	+ 42
†6553	18 3.2	−25 56	332	− 3.4	320	320	− 19
‡6558	18 3.8	−31 47	327	− 6	455	450	− 48
*6569	18 7.2	−31 51	328	− 7	400	395	− 49
†6760	19 6.1	+ 0 52	3	− 4.6	525	525	− 42
*7492	23 3.1	−16 10	21	−64	285	125	−255

* Certainly a globular cluster.

† Almost certainly a globular cluster.

‡ Probably a globular cluster but as yet not proved.

Herschels or earlier observers more than eighty years ago. This is further evidence of the completeness of our lists of globular clusters. (3) N.G.C. 7006, with adopted distance of 67,000 parsecs, still holds its place as the most remote sidereal object of definitely estimated distance.

Further observations are likely to prove that N.G.C. 6355 and 6535 are globular, but present photographs of them are hardly conclusive. The other three unproved systems are too far south for the 60-inch reflector.

Perhaps the most striking result of this special survey is the evidence that every faint, little-condensed cluster in galactic latitude higher than 15° or 20° is really globular, although short exposures and visual observations had in several cases heretofore recorded few stars. On the other hand, the similar faint clusters along the galactic equator, without exception, are open groups with no condensed background of faint stars appearing on long exposures. N.G.C. 7492, for instance, was formerly considered an exception[1]—an open cluster outside the galactic segment—but it is actually globular, containing thousands of stars. The evidence grows continually stronger that open and globular clusters occupy regions of space that are mutually exclusive.

There is also some evidence that an abnormal type of globular cluster exists, one in which the brighter stars are fainter and more scattered than is usually the case. In their luminosity-curves a distinct break appears to occur between the brighter and fainter stars, and for such systems the parallax-diameter relation may not be strictly applicable. The known examples of this type are N.G.C. 5466, 6366, and 7492; it might be well also to place N.G.C. 4372, 5897, and 6144 in this group, although for two of them an alternative interpretation of the apparent discrepancy between diameter and magnitude may be available.[2] The abnormal form possibly represents an early or late stage, or a disturbed condition of a typical globular cluster. In N.G.C. 7492, for which some preliminary colors are available, the brightest stars are yellow.

III. ON THE DISTRIBUTION IN SPACE OF 86 GLOBULAR CLUSTERS

For the reasons apparent from the following remarks it seems worth while to revise the plots and extend the discussion of the distribution in space of globular clusters. Fig. 3 shows the projection on a plane perpendicular to the Galaxy and oriented to include galactic longitude 325°. It gives the appearance of the system of clusters as seen from longitude 235° and is thus merely a revision

[1] *Publications of the Astronomical Society of the Pacific*, **30**, 50, 1918.

[2] *Mt. Wilson Contr.* No. 152, p. 14, n. 2, 1917.

of Fig. 4 of the seventh paper of this series.[1] The values of the abscissae, $R \cos\beta \cos(\lambda - 325°)$, were determined graphically for

TABLE III

SPACE CO-ORDINATES OF GLOBULAR CLUSTERS

N.G.C.	$R \sin\beta$	$R \cos\beta \cdot \cos(\lambda-325°)$	$R \cos\beta \cdot \sin(\lambda-325°)$	N.G.C.	$R \sin\beta$	$R \cos\beta \cdot \cos(\lambda-325°)$	$R \cos\beta \cdot \sin(\lambda-325°)$
104....	− 47	+ 29	− 39	6341....	+ 69	+ 33	+ 96
288....	−189	− 3	− 7	6352....	− 28	+216	− 66
362....	−109	+ 58	− 89	6355*...	+ 38	+500	+ 18
1261....	−199	+ 8	−161	6356....	+ 67	+375	+ 59
1851....	− 96	− 58	−131	6362....	− 38	+105	− 66
1904....	−120	−145	−173	6366....	+ 89	+290	+111
2298....	− 63	− 88	−219	6388....	− 34	+269	− 62
2808....	− 32	+ 40	−162	6397....	− 17	+ 76	− 29
3201....	+ 26	+ 23	−143	6402....	+ 56	+207	+ 92
4147....	+514	− 23	−107	6426....	+150	+471	+283
4372....	− 18	+ 63	− 94	6440....	+ 23	+516	+100
4590....	+ 97	+ 70	−108	6441....	− 40	+452	− 32
4833....	− 23	+ 96	−132	6496*...	+ 58	+296	− 52
5024....	+186	+ 34	− 11	6517....	+ 55	+579	+234
5139....	+ 18	+ 41	− 46	6535*...	+ 64	+319	+177
5272....	+136	+ 20	+ 21	6539....	+ 42	+368	+156
5286....	+ 37	+134	−139	6541....	− 28	+142	− 22
5466....	+180	+ 42	+ 42	6553....	− 19	+318	+ 39
5634....	+229	+191	− 55	6558*...	− 48	+450	+ 16
I.C. 4499	− 85	+148	−183	6569....	− 49	+395	+ 21
5897....	+ 75	+125	− 31	6584....	− 73	+243	− 70
5904....	+ 90	+ 86	+ 11	6624....	− 40	+282	+ 25
5927....	+ 16	+159	− 95	6626....	− 19	+181	+ 32
5946*...	+ 29	+359	−208	6637....	− 41	+209	+ 15
5986....	+ 47	+191	− 69	6638....	− 48	+337	+ 60
6093....	+ 65	+188	− 16	6652....	− 65	+305	+ 16
6101....	− 55	+156	−135	6656....	− 12	+ 80	+ 18
6121....	+ 31	+109	− 13	6681....	− 41	+177	+ 12
6144....	+ 63	+235	− 25	6712....	− 32	+274	+145
6171....	+ 60	+148	+ 16	6715....	− 44	+153	+ 22
6205....	+ 71	+ 40	+ 75	6723....	− 39	+121	+ 4
6218....	+ 52	+106	+ 34	6752....	− 39	+ 73	− 30
6229....	+274	+ 82	+328	6760....	− 42	+414	+323
6235....	+112	+487	+ 8	6779....	+ 30	+105	+225
6254....	+ 45	+106	+ 32	6809....	− 41	+ 90	+ 16
6266....	+ 19	+150	− 13	6864....	−206	+376	+152
6273....	+ 25	+157	− 3	6934....	−114	+180	+256
6284....	+ 64	+364	0	6981....	−164	+192	+150
6287....	+ 83	+428	+ 15	7006....	−228	+245	+577
6293....	+ 37	+261	0	7078....	− 69	+ 49	+121
6304....	+ 28	+320	− 11	7089....	− 94	+ 70	+104
6316....	+ 46	+524	0	7099....	−128	+101	+ 56
6333....	+ 43	+244	+ 34	7492....	−255	+ 70	+104

* Probably a globular cluster but not definitely proved.

[1] *Mt. Wilson Contr.* No. 152, 1917. The point for M 75 (N.G.C. 6864) is erroneously plotted in the earlier figure.

the earlier plot; they are now computed and entered in the third column of Table III. All globular clusters are represented in the new diagram, including the five unproved objects marked with the double dagger in Table II. The open circles designate clusters for which the provisional distances are marked in Table II by a colon.

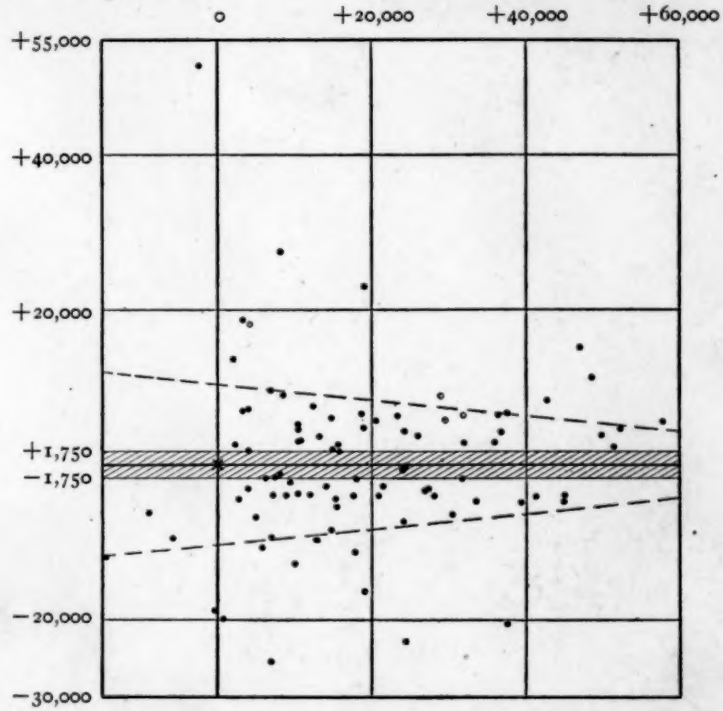

FIG. 3.—Projection of the complete system of globular clusters on a plane perpendicular to the Galaxy and oriented to include the line from the sun to the center of the system. Ordinates are $R \sin \beta$; abscissae are $R \cos \beta \cos (\lambda - 325°)$; unit of distance is one parsec. The sun at the origin of co-ordinates is marked by a cross. See Fig. 4 of *Mt. Wilson Contr.* No. 152.

The parallax of Messier 22 (N.G.C. 6656) has been increased slightly over the value previously adopted. The cluster is fairly open and in a rich field, and it is now found that the stars selected for the study of magnitudes were so near the center that an error of 0.15 mag., due to the Eberhard effect, crept into the earlier results. The values of the galactic co-ordinates for some of the

clusters in Table V of the seventh paper were taken from the catalogues of Bailey, who used a position of the North Galactic Pole slightly different from that adopted for this work. The computations in Table III are all based on the co-ordinates in Melotte's catalogue, with the result that the new values of $R \sin \beta$ differ in a few cases from those previously computed. The revised galactic positions, however, never differ by more than a degree in either coordinate, except for the longitudes in high galactic latitude.

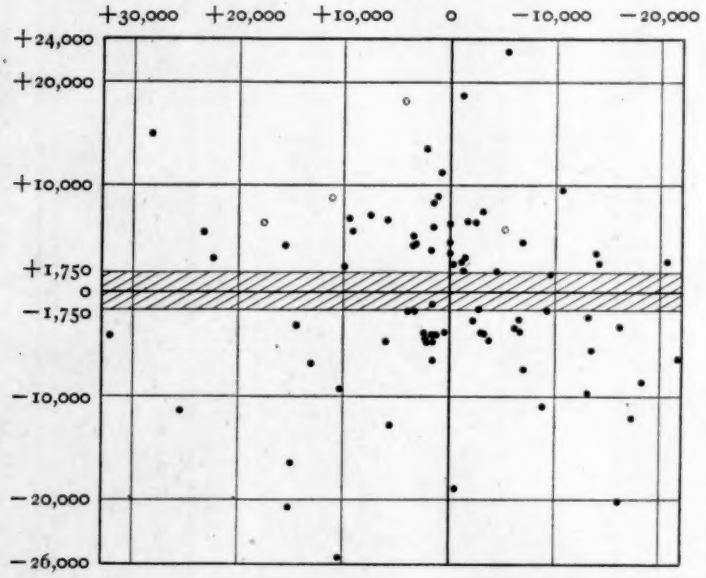

FIG. 4.—Projection of the system of globular clusters on a plane perpendicular to the Galaxy and to the plane of the preceding diagram. Ordinates are $R \sin \beta$; abscissae are $R \cos \beta \sin (\lambda - 325°)$.

Three points formerly not definite are now emphasized by Fig. 3: (1) There is no sensible increase in the minimum distance from the galactic plane with increasing distance from the sun. (2) The additional results have filled in the gaps of the earlier work and show that the provisional estimate of a distance of 20,000 parsecs to the center of the system is not too great. (3) There is evidence that the globular clusters occupy a space shaped somewhat like a split wedge, the base of which passes nearly through the

sun and contains the great circle defined by galactic longitudes 55° and 235°. Only a part of the tapering of the wedge beyond the center of the system is to be attributed to a lack of observations. The inclined broken lines in Fig. 3 suggest the degree of tapering away from the extremely broad and sharply defined base.

Fig. 4 shows the projection of globular clusters on a plane perpendicular to the Galaxy and to the plane of Fig. 3. It represents the appearance of the system of clusters as seen from a great distance in longitude 145°, indicating approximately the apparent distribution as seen from the earth. The co-ordinates for this plot are in the second and fourth columns of Table III. The close approach to symmetry when viewed from this angle is interesting; the numbers of clusters in the four quadrants are 20, 23, 23, and 20. N.G.C. 4147, 6229, and 7006 are outside the limits of the figure.

It is a striking fact that the system of 86 globular clusters listed in Table III is divided into exactly equal numbers by the plane of the Milky Way.

IV. NOTE ON THE ABSENCE OF GLOBULAR CLUSTERS FROM MID-GALACTIC REGIONS

In the seventh and twelfth papers of this series[1] comments have been made upon the absence of globular clusters from the equatorial region of the galactic system. The importance of the phenomenon necessitates a full consideration of its reality and meaning. That the condition is real is attested by such evidence as: (1) the consistent agreement of the results for clusters at various intervals of distance along the galactic plane; (2) the presence of blue stars and open clusters on the galactic equator at distances equal to those of the nearer globular clusters; (3) the absence of appreciable light-scattering in space; and, finally, (4) the apparent dynamical relation of globular to open clusters and their complementary distribution.

Probably the best evidence that dark nebulosity does not obscure globular clusters in the equatorial segment (the nebulous clouds assumed, for instance, to be analogous to the peripheral

[1] *Mt. Wilson Contr.* No. 152, p. 22, 1917; No. 157, pp. 6, 10, 1918.

ring of absorbing matter observed in some spiral nebulae) is afforded by the diagram of Fig. 5. Let us consider first the 31 globular clusters for which the distance projected on the plane of the Milky Way, $R \cos \beta$, does not exceed 15,000 parsecs. Fifteen are north of the galactic plane and sixteen are

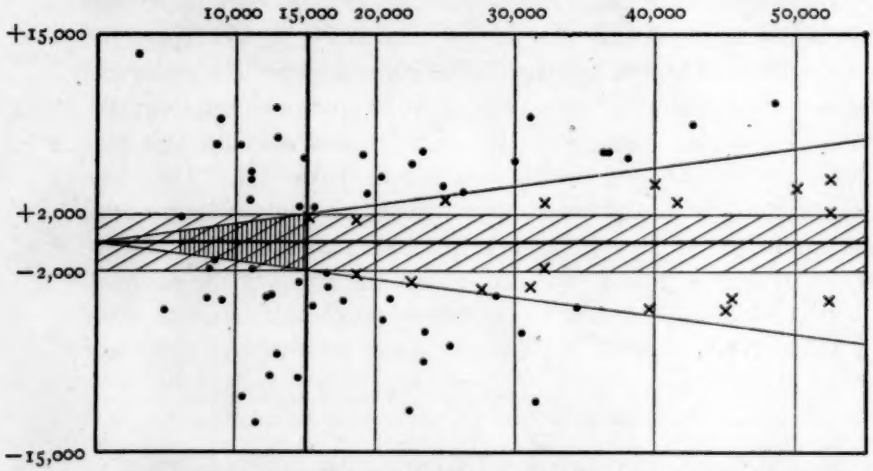

FIG. 5.—Diagram to illustrate that the equatorial segment may not be devoid of globular clusters because of clouds of obstructing matter in the Milky Way. Ordinates are $R \sin \beta$; abscissae are $R \cos \beta$; the unit of distance is one parsec. Twelve clusters fall outside the limits of this diagram.

south. Their frequency in distance from the plane, $R \sin \beta$, is as follows, the unit of distance being 100 parsecs:

$R \sin \beta$	$> \pm 200$	± 200 to ± 150	± 150 to ± 100	± 100 to ± 75	± 75 to ± 50	± 50 to ± 25	$< \pm 25$	-12 to $+18$
Number	3	2	3	5	5	9	4	0

This tabulation, as well as the part of the diagram to the left of the heavy vertical line, shows definitely the absence from the Milky Way of the nearer clusters.

If we attribute the absence of globular clusters to the obscuration by dark matter in low latitudes, we may indicate this hypothetical obscuring material by the vertical shading across the equatorial segment. If the nearby clusters are missing because of these clouds of obstructing matter, then certainly the faint and more distant clusters behind the clouds should be wanting. That

is, the light from every visible cluster with galactic latitude less than $\pm 8°$ would pass through this region, which, by supposition, is capable of concealing twenty or thirty systems that are relatively near. To maintain the assumption, no clusters should be found within the diverging lines. Observation shows nearly a score. The supposition, therefore, that the mid-galactic regions are not transparent appears untenable.

During the six months that have elapsed since the preceding paragraphs were written some doubt has been thrown on the foregoing conclusion, notwithstanding the amount and character of the evidence in its favor. A discussion of the new arguments and the observational data will be given elsewhere in connection with a treatment of the parallaxes of open clusters. For the present the reality of the avoidance of the galaxy by globular clusters must be considered an open question.

We cannot suppose that the 19 clusters[1] within the diverging lines have been estimated too distant because of partial obstruction of their light, and that they are actually the clusters missing from the nearer equatorial segment. The distances of these clusters are based upon angular diameters, some upon magnitudes as well, and we have found that the parallax-diameter relation holds whether the clusters are near or far from the galactic plane.[2]

V. ON THE PARALLAX-DIAMETER RELATION FOR GLOBULAR CLUSTERS

The relation between the distances, as determined both from variable stars and from mean magnitudes, and the apparent diameters, as measured on the Franklin-Adams charts, is unexpectedly definite, if we consider the various constitutional differences among globular clusters.[3] The relatively small deviations from the parallax-diameter curve[4] appear to mean that abnormal clusters are rare. That the correlation of distance and apparent diameter is

[1] One cluster with $R \sin \beta = +5500$ parsecs falls outside the diagram to the right.

[2] In the sixth section of this paper the additional parallax-diameter curves are definite, notwithstanding the large differences in exposure-time and in the brightness of the stars appearing in the photographs.

[3] For instance, the occasional contrast in concentration of the brighter stars, noted in the second section of this paper.

[4] Fig. 1 of *Mt. Wilson Contr.* No. 152.

actually fundamental, however, and is therefore reliably applicable to the estimation of parallaxes not otherwise obtainable, is further attested by the supplementary data represented in the curves of Fig. 6, which we shall presently describe.

The significance of these fairly uniform results must be that, for the large majority of globular clusters, the linear dispersion of the central nucleus[1] is nearly constant, although we cannot as yet decide definitely whether that condition connotes approximately simultaneous origin of all known extra-galactic cluster systems, or rather an essentially permanent dynamical condition in spheroidal stellar groups. The second alternative is supported by the evidence of numerous open galactic clusters. Such groups appear to maintain dimensions of the same order as those of globular systems, but they show, in the scarcity of their red giants and the preponderance of highly luminous blue stars, indications either of much greater age or of more expeditious development.

Without doubt the most homogeneous photographic survey now available for the whole sky is that initiated by Mr. Franklin-Adams;[2] the uniform length of exposure, the quality of stellar images, and the scale of the photographic charts are all particularly suitable for the study of the composite images of clusters, reported in *Mount Wilson Contributions*, No. 152.

We have a second comprehensive photographic survey in the Harvard Map of the Sky, but the copies (positive prints on glass) of the originals show considerable lack of homogeneity in quality and limiting magnitude;[3] in particular, the scale of the plates is so small (aperture 1 inch, focal length 13 inches, exposures 39 to 75 minutes) that high accuracy cannot be expected in determining the apparent diameters of the nearly starlike images of globular clusters.[4] Notwithstanding the difficulties due to faintness, small

[1] The estimates of diameter on the Franklin-Adams charts "refer actually to what appears to be a central core of each system. The scale of the photographs does not permit close differentiation of the outlying members of a cluster from the stars of its surrounding field," p. 25, n. 1, *Mt. Wilson Contr.* No. 152.

[2] *Memoirs of the Royal Astronomical Society*, **60**, Part 3, 1915.

[3] Cf. Nort, *Recherches Astronomiques de l'Observatoire d'Utrecht*, VII, 1917.

[4] The Harvard Map is described in *Harvard Circular*, No. 71, 1903.

size, and the resulting confusion with the surrounding fields, the mean results show, in the upper curve of Fig. 6, a definite progression of apparent diameter with adopted parallax. Each point represents the mean of five values. Many of the most distant clusters could not be certainly identified on this scale; and ω Centauri, with diameter 22′.6, parallax 0″.00015, is also not plotted. The deviations for the individual clusters average nearly 25 per cent of

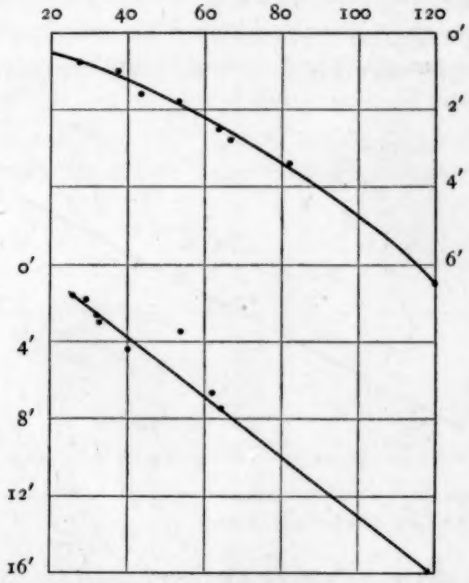

FIG. 6.—Parallax-diameter curves for globular clusters. Above: diameters from plates of Harvard Map of the Sky; below: diameters from Palisa-Wolf charts; ordinates are diameters; abscissae are parallaxes in units of 0″.000001.

the adopted parallaxes, a decidedly lower accuracy than shown by results from the Franklin-Adams charts[1] but still of considerable value as a further justification of the method and as a check of the separate values.

The photographs of the Milky Way by Bailey[2] are also serviceable for this work, since many of the globular clusters lie within 10° of the galactic circle. The scale of his photographs is about the

[1] Cf. Table VII of *Mt. Wilson Contr.* No. 152.

[2] *Harvard Annals*, **72**, No. 3, 1913; **84**, No. 4, 1916.

same as for the Harvard Map, and a casual examination gives comparable results.

The scale of the photographic charts published by Palisa and Wolf is more than double that of the Franklin-Adams charts, but the series is incomplete and the exposure times for the few plates that contain clusters vary from 1^h40^m to 3^h30^m. The results for the nine clusters available are in the lower part of Fig. 6; they suggest the high value of a complete series of long exposures on the larger scale, and they again emphasize the validity of this manner of estimating distance. The discordant point refers to N.G.C. 6626, a compact cluster in declination $-25°$.

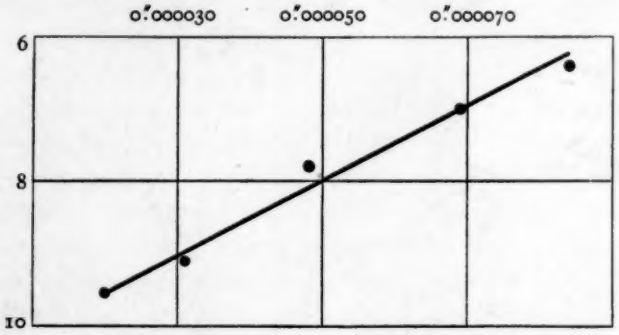

FIG. 7.—Parallax and integrated visual magnitude. Ordinates are Holetschek's magnitudes; abscissae are adopted parallaxes.

Nearly all of the values of diameter used in this section are based upon two independent series of measures by Miss Davis.

VI. TOTAL LIGHT AS A MEASURE OF THE DISTANCES OF CLUSTERS

The mean apparent magnitude of the 25 brightest stars in a globular cluster has been used with satisfactory results as a criterion of distance.[1] The integrated apparent brightness of the same stars, or even of the hundred or so brightest stars in each system, would probably be an equally good measure of the distance. The integrated light of all the thousands of stars in a globular cluster, however, does not greatly exceed that due to its brightest stars alone;

[1] See the fifth section of the sixth paper of this series, *Mt. Wilson Contr.* No. 151, 1917.

and if it were possible, after overcoming some of the obvious minor difficulties, to measure accurately the total apparent brightness of such systems, possibly a valuable method would result for deriving the relative parallaxes of all typical globular groups.

A study has been made by Holetschek[1] of the integrated visual magnitudes of the brighter clusters and nebulae visible at Vienna. His adopted magnitudes for globular clusters, given in Table IV, are of very unequal merit owing to disparity of observation, varying diffuseness of images, and availability of comparison stars. Magnitudes of the comparison stars were taken in general from the B.D.

TABLE IV

HOLETSCHEK'S INTEGRATED MAGNITUDES OF GLOBULAR CLUSTERS

N.G.C.	Mag.	Parallax	N.G.C.	Mag.	Parallax	N.G.C.	Mag.	Parallax
1904.....	8.0	39	6229....	8.6	23	6637....	9.0	47
4147.....	9.4	19	6235....	9.7	20	6656....	6.2	118
4590.....	8.2	62	6254....	6.9	83	6681....	9.5	55
5024.....	7.8	53	6266....	7.0	66	6712....	8.9	32
5272.....	6.6	72	6273....	6.8	63	6760....	10.5	19
5466.....	8.5	51	6284....	9.5	27	6779....	8.3	40
5634.....	9.7	33	6287....	9.2	23	6864....	8.0	22
5897.....	10.2	67	6293....	8.5	38	6934....	9.0	30
5904.....	6.7	80	6333....	7.3	40	6981....	9.5	34
6093.....	7.8	50	6341....	6.2	81	7006....	9.7	15
6121.....	6.8	88	6356....	8.5	26	7078....	6.2	68
6171.....	9.0	62	6402....	7.8	43	7089....	6.7	64
6205.....	5.8	90	6626....	7.9	54	7099....	8.5	58
6218.....	6.8	81						

catalogues, and high accuracy is not claimed for the results. The parallaxes in Table IV are the adopted values from *Mount Wilson Contributions*, No. 152; the unit is 0".000001.

Omitting the magnitudes for clusters south of declination $-20°$ because of uncertainties connected with low altitude and short observing season, we have combined the remaining twenty-three values into normal groups in order of parallax. The plot of the normal points in Fig. 7 shows the expected uniform decrease of brightness with increasing distance. Obviously a systematic comparison with recognized magnitude standards, using a telescope of

[1] *Annalen der k.k. Universitäts-Sternwarte in Wien*, 20, 114, 1907.

extremely short focus, or a method that is independent of the angular diameters of the images compared, would contribute important material to the problem of the relative parallaxes of star clusters.

VII. THE CONTRASTED MOTIONS AND DISTRIBUTION OF GLOBULAR CLUSTERS AND SPIRAL NEBULAE

The known radial velocities of globular clusters are predominantly negative.[1] These enormous stellar systems, moving under gravitational attraction with an average speed of more than 100 km/sec., are apparently as a class approaching the sun; and, remembering their extra-galactic positions, we also infer that they are approaching and falling into the dense stellar strata of the general galactic system. To be sure, the spectroscopic study of clusters has not gone far as yet, but the foregoing inference does not rest upon observed velocities alone. The composition of the galactic system and the distribution of clusters in space, especially the relation of globular systems to the open groups of mid-galactic regions,[2] yield more important evidence than is afforded by radial velocities that the galactic system absorbs globular clusters.

On the other hand, the brighter spiral nebulae as a class, apparently regardless of the gravitational attraction of the galactic system, are receding from the sun and from the galactic plane—a remarkable condition that has been little emphasized heretofore. From the published spectroscopic work of Adams, Campbell, Moore, Paddock, Pease, Sanford, Wolf, Wright, and particularly of V. M. Slipher at Flagstaff, we obtain the radial velocities in the fifth column of Table V. The Andromeda nebula (N.G.C. 224) and its companion are treated as a single object. The radial velocity of the nucleus of Messier 33, as determined by Pease from absorption lines, is given in preference to the earlier values derived by Pease and Slipher from bright lines.[3]

Limited as this material is, it yields some important results, which emphasize the contrast between clusters and spiral nebulae

[1] See *Mt. Wilson Contr.* No. 157, 1918, Table I and Section 6.

[2] Sections I and III of this paper.

[3] *Publications of the Astronomical Society of the Pacific*, **28**, 33, 1916.

and bear directly upon the structure and present status of the galactic system. These preliminary results, appearing in the seven numbered divisions below, are the more worthy of mention at the present time because the increase of observational data for spirals will necessarily be slow from now on, due to the extreme faintness of the nebulae not already observed for radial velocity.

TABLE V

POSITIONS AND RADIAL VELOCITIES OF 25 SPIRAL NEBULAE

N.G.C.	Messier	Galactic		Radial Velocity	Apical Distances, θ				Vis. Mag. (Holetschek)
		Long.	Lat.		$\omega=90°$	$\omega=105°$	$\omega=120°$	$\omega=150°$	
				km					
224	31	89°	−20°	− 316	20°	25°	36°	63°	5.0
598	33	102	−30	− 70	32	30	35	55	7?
1023	.....	112	−19	+ 300	29	20	21	42	9.7
1068	77	141	−52	+1120	67	60	55	52	8.7
2683	.....	158	+40	+ 400	73	63	53	41	9.2
3031	81	109	+42	− 30	45	42	43	56	8.0
3115	.....	216	+38	+ 600	118	106	95	71	9.0
3379	.....	218	+59	+ 810	108	102	94	79	9.1
3521	.....	225	+54	+ 730	115	107	99	81	9.3
3623	65	209	+64	+ 800	102	96	90	77	8.9
3627	66	211	+64	+ 650	103	97	90	78	8.6
4151	.....	118	+76	+ 940	78	76	76	78	10.7
4258	.....	103	+68	+ 500	69	68	69	75	8.7
4526	.....	262	+71	+ 580	109	107	105	97	10.0
4565	.....	215	+88	+1100	91	91	90	89	9.4
4594	.....	267	+52	+1180	128	126	121	106	8.7
4649	60	265	+75	+1090	105	104	102	96	8.6
4736	94	76	+86	+ 290	86	86	87	89	7.7
4826	64	295	+84	+ 150	95	96	96	95	8.6
5005	.....	64	+78	+ 900	79	81	83	89	9.1
5055	63	69	+74	+ 450	75	77	80	88	9.2
5194	51	68	+71	+ 270	72	75	78	87	8.4
5236	83	283	+31	+ 500	147	148	145	126	9.5
5866	.....	59	+52	+ 650	58	65	73	91	10.3
7331	.....	62	−22	+ 500	35	47	61	88	9.3

1. Since the greater number of bright spirals are north of the galactic plane and their positions are more favorable for northern observers, only five negative galactic latitudes appear in Table V. To these should be added that of N.G.C. 1700, for which Sanford estimates a large positive velocity, though, in the absence of a definite numerical value, it is not included in the table. Only one of the twenty velocities for spirals north of the plane is negative,

while two of the spirals on the south side are approaching. But the three negative values depend, as we shall see, on low galactic latitude and high apparent brightness rather than on the sign of the latitude, and hence we conclude that essentially without exception, on both sides of the Galaxy, spiral nebulae recede.

2. The speed of spiral nebulae is dependent to some extent upon apparent brightness, indicating a relation of speed to distance or, possibly, to mass. The six spirals with smallest radial velocity, including all of those with negative values, are not exceeded in brightness by any spirals in Holetschek's list of visual magnitudes:[1]

N.G.C.	224	598	4736	3031	5194	4826
Integrated magnitude	5.0	7?	7.7	8.0	8.4	8.6
Radial velocity	−316	−70	+290	−30	+270	+150 km

The arithmetical mean of these six velocities is ± 188 km/sec.; their algebraic mean is +49 km/sec., while for the other 19 spirals of Table V it is fifteen times as large—that is, +726 km/sec. Only three spirals besides these six bright ones are now known to have radial velocities of less than +500 km/sec.

3. Forming means of five in order of galactic latitude, we derive from Table V the first two lines of the following tabulation, which indicate that speed may be related to angular distance from the galactic equator:

Mean galactic latitude, β	24°	45°	59°	72°	82°
Mean radial velocity, V_r	+183	+548	+834	+578	+676 km
$V_p = V_r \operatorname{cosec} \beta$	+450	+775	+975	+610	+680 km

If we should assume that the motion is wholly perpendicular to the galactic plane, the mean velocities would be as given in the last tabulated line above.

4. The correlation of velocity and the latitude co-ordinate, although not very definite, may be of some significance for theories of the spiral nebulae; but, guided by the provisional hypothesis described in Section VIII of this paper, we find some evidence of a more striking relation between the velocities of spirals and a new position co-ordinate. Let λ and β denote the galactic

[1] *Annalen der k.k. Universitäts-Sternwarte in Wien*, **20**, 114, 1917. N.G.C. 4826 is equaled in brightness by two other spirals, according to Holetschek's estimates.

co-ordinates of any spiral, and ω the longitude of an origin on the galactic circle. Then the angular distance, θ, of the spiral from this origin is given by

$$\cos\theta = \cos\beta\cos(\lambda-\omega) \qquad (1)$$

In the last four columns of Table V we give for each spiral the values of θ for $\omega = 90°$, $105°$, $120°$, $150°$. The angle θ may lie between $0°$ and $180°$, and at the galactic poles is $90°$; in the table, however, no value of θ is less than $20°$ because of the avoidance of the Milky Way by spiral nebulae, and few values exceed $120°$ because the most southern nebulae have not been observed for velocity.

TABLE VI

THE PROGRESSION OF THE MEAN VELOCITY OF SPIRAL NEBULAE WITH DISTANCE FROM THE GALACTIC APEX

INTERVAL OF θ	$\omega = 90°$			$\omega = 105°$			$\omega = 120°$		
	Mean θ	Mean V_r	Number	Mean θ	Mean V_r	Number	Mean θ	Mean V_r	Number
$\leqq 50°$ ..	32°	+ 77	5	33°	+ 77	5	34°	− 29	4
51°– 75 ..	69	+565	6	66	+588	5	62	+634	5
76 –100 ..	86	+676	5	88	+660	8	88	+641	12
101 –125 ..	109	+751	7	105	+762	5	109	+950	3
>125 ..	138	+840	2	137	+840	2	145	+500	1

For the first three values of ω the progression of the observed radial velocity of spirals, V_r, with increasing θ is shown in Table VI for equal intervals of θ. The interval for θ less than $50°$ contains in all cases the three bright nebulae with negative radial velocities, and the mean V_r is correspondingly affected. The correlation, if real, is about equally definite for $\omega = 90°$ and $\omega = 105°$. Its meaning would be that, regardless of galactic latitude, the average radial velocity of spiral nebulae increases with the angular distance from a point in the northern Milky Way—a point which, in anticipation of an explanation proposed in Section VIII, we may call the galactic apex.

In these progressions we have a suggestion that average radial velocity may be roughly predicted from position; but before this relation can be definitely established from spectroscopic results alone, we must have more data, for the wide range of peculiar

velocity in Table V is inadequately reflected by the means of Table VI. Other groupings could alter or even conceal the uniform progression of mean velocities.

If we omit the two brightest nebulae for the sake of greater homogeneity (see p. 126) and combine the others into four groups in order of increasing θ, we obtain for $\omega = 105°$:

Mean θ	46°	74°	95°	116°	(2)
Mean V_r	+458	+618	+633	+780 km	
Number of spirals	5	6	6	6	

5. On the basis of this more homogeneous material we may make the following computation, which is perhaps to be considered more as an interesting illustration than as an approach to definite cosmic fact. Let V_s be the average systematic recessional velocity of spiral nebulae in the line of sight, and let V_g be a quantity which we may call the velocity of the galactic system toward $\lambda = 105°$, $\beta = 0°$. Then

$$V_s - V_g \cos\theta = V_r \tag{3}$$

and from the values of θ and V_r in (2) we obtain, provisionally,

$$\begin{aligned} V_s &= +650 \text{ km/sec.} \\ V_g &= +300 \text{ km/sec.}^1 \end{aligned} \tag{4}$$

Employing these provisional values of V_s and V_g, and setting up conditional equations of the form (3) for each of the 23 spirals in the tabulation (2), we derive the following normal equations:

$$\begin{aligned} 23\Delta V_s - 1.96\Delta V_g - 120 &= 0 \\ -1.96\Delta V_s + 4.34\Delta V_g + 80 &= 0 \end{aligned}$$

The solution gives for corrections to (4), $\Delta V_s = +8$ km/sec., $\Delta V_g = +17$ km/sec. The adopted values with their probable errors are

$$\begin{aligned} V_s &= +660 \pm 45 \text{ km/sec.} \\ V_g &= +320 \pm 100 \text{ km/sec.} \end{aligned}$$

The average difference between an observed velocity and that computed by putting the foregoing values in formula (3) is ± 240

[1] By assuming $V_g = +300$ km/sec., the mean value of V_s derived from *all* the material of Table VI is +635 km/sec. for $\omega = 105°$, in close agreement with the result above.

km/sec., a quantity that is to be taken as representing, not observational errors, but rather the peculiar velocities of spirals.

From the foregoing result we infer that one way of accounting for the observed increase of average V_r with distance from the so-called galactic apex is by assuming that the galactic system moves toward longitude 105°, approximately, with a velocity of some three hundred kilometers a second, while the spirals of the brightness here involved recede with an average systematic radial velocity of six or seven hundred kilometers a second.[1]

6. If we should assume that the average systematic motion is perpendicular to the galactic plane rather than radial from the center of the galactic system, or radial from the sun as is observed and assumed above, then we would have for this perpendicular velocity

$$V_p = (V_r - V_g \cos\theta) \operatorname{cosec} \beta$$

and a solution of the four mean observational equations derivable from (2) gives, provisionally,

$$V_p = +775 \text{ km/sec.}, \quad V_g = +140 \text{ km/sec.}$$

The least-squares solution of the 23 conditional equations then leads to the following results:

$$V_p = +765 \pm 55 \text{ km/sec.}$$
$$V_g = +110 \pm 100 \text{ km/sec.}$$

and the average value of O − C is ± 250 km/sec. The probable errors give little choice between assumptions involving radial and perpendicular systematic motion, but the conception of radial motion is distinctly preferable from the standpoint of physical probability.

When sufficient data become available, not only should V_g and V_s (or V_p) enter the computations, but also the co-ordinates of the origin of θ.

[1] Following the procedure commonly used to determine the apex of the solar motion, Truman, Young and Harper, and Slipher have computed from the radial velocities of spirals a motion of the galactic system toward a rather vaguely defined southern apex; but they have assumed, explicitly, a random peculiar motion for spiral nebulae and, implicitly, the "island universe" hypothesis. Their result is an obvious consequence of the preferential recession and of the absence of spirals of known velocity from the southern sky.

7. From the present evidence we see that there is a magnitude effect for the very brightest objects; in addition the observed velocities of spirals may be divided into three parts, the first two of which appear to be very definite: (*a*) a systematic radial recession of more than 600 km/sec., which is increased by one-sixth if the systematic motion is assumed to be perpendicular to the galactic plane; (*b*) the peculiar velocities, which average about ± 250 km and are represented by the deviations from the means and formulae; (*c*) a component of the radial velocity whose effect decreases on the average with distance from the "galactic apex," and which may be interpreted as a drift of the whole galactic system with respect to the brighter spiral nebulae. This last component is the least definite, quantitatively, but on the other hand we shall see that its existence seems to be affirmed by the distribution of spirals and by other considerations discussed in the last part of Section VIII.

The apparent distribution of spiral nebulae appears to be sufficiently known for a general statement, though much remains to be done in extending the recent work of Hardcastle, Fath, Sanford, and Curtis. That the spirals approach nearest to the Milky Way in two hours of right ascension ($\omega = 100°$) has been noted by Hinks, Stratonoff, and others. Globular clusters are wholly absent from that region, and it is very significant that in the opposite region of the sky, where globular clusters and clouds of stars most abound, the avoidance of the Milky Way by spirals reaches its maximum.[1]

Thus the region avoided, at least by the brighter spirals, is roughly defined by a wedge, symmetrical in relation to the galactic plane, with its base in the general direction now adopted as the center of the galactic system. A wedge-shaped arrangement (analogous to the wedge-shaped avoidance by spirals) has been pointed out for globular clusters in Section III of this paper, but the base of the wedge lies in a direction approximately opposite to the direction of the center.

[1] See the diagrams by Hardcastle and Hinks (*Monthly Notices*, **74**, 699, 1914) and the bibliography and discussion by Sanford (*Lick Observatory Bulletins*, **9**, 82, 1917).

There is, accordingly, in the contrasted distribution of spirals and globular clusters, as well as in the opposing directions of their motions, an indication that the compelling forces act in opposite directions.

VIII. DATA AND INFERENCES FOR A PROVISIONAL COSMOGONY

The observational results discussed in the foregoing sections of the present contribution and in the thirteen preceding papers of this series permit the statement of five of the conditions that must be considered in attempting to account for the origin of the galactic system and its present relation to clusters and spiral nebulae: (1) the similarity of globular clusters in dimensions and content; (2) the complementary distribution of open and globular clusters; (3) the existence of thousands of suborganizations in the galactic system; (4) the contrasted distribution of spiral nebulae and globular clusters; and (5) the opposed directions of preferential radial motion for spirals and clusters.

In the following paragraphs we offer a brief summary of the observational data bearing on each of these five conditions and a suggestion as to the general interpretation of the evidence.

1. The parallax-diameter curves in *Mount Wilson Contributions*, No. 152 and in Section V of this paper indicate that with few exceptions globular clusters have approximately the same linear diameters. Observations of (*a*) the integrated light of clusters, (*b*) their general luminosity-curves, and (*c*) the phenomena of color and absolute magnitude of their giant stars show also that the stellar content is much the same from system to system.

2. For globular clusters the frequency-curve of galactic latitudes has a distinct minimum at the galactic plane, whereas the corresponding curve for open clusters shows a pronounced maximum in low latitudes (Figs. 1 and 2 of this paper). The evidence suggests that the two kinds of clusters occupy regions of space that are mutually exclusive (Sections I and III of this paper). There is some indication that a transition from one kind to the other occurs along the outskirts of the equatorial galactic segment.

3. Numerous conditions suggest that the Galaxy is a heterogeneous assemblage of unequally organized parts. The remarkably

wide pairs of stars of common motion, the local moving groups, the open clusters, the clouds of the Milky Way, the star-streams[1]—all these combinations indicate that the galactic system may be largely composed of disintegrating clusters. The testimony of the distribution and radial motion of globular clusters indirectly supports this view, and suggests further that the system is now growing and has gradually grown throughout the past from a much less complicated state.

4. While both globular clusters and spiral nebulae appear to be mainly if not wholly outside the equatorial galactic segment, they occur in general in different parts of the sky. In the Southern Hemisphere the globular clusters crowd in close to the Milky Way and the bright spirals widely avoid it; in the Northern Hemisphere the spirals approach their nearest to the galactic circle and globular clusters are almost wholly absent.[2]

5. Globular clusters as a class appear to be rapidly approaching the galactic system; spiral nebulae as a class are receding with high velocities. The relation of the velocities of spirals to brightness and to position in the sky has been dealt with above in Section VII. Some of the reasons for not considering spiral nebulae to be separate galactic systems have been outlined in the introduction to the twelfth paper of this series. For the present we shall accept that the distances of globular clusters and spirals are of the same order, and that, with the possible exception of a few of the very brightest, none is within mid-galactic stellar regions.

The foregoing conditions, when considered in connection with previously accepted stellar and nebular results, suggest as a preliminary hypothesis that the discoidal galactic system originated

[1] Accepting the evidence presented in *Mt. Wilson Contr.* No. 157 (see also the ninth section of this contribution) bearing on the existence of a local cluster, we note that from material now available the circum-solar cluster appears to be both larger and more oblate than typical open or globular systems; it seems to be more like the Magellanic Clouds in dimensions. Possibly before it became a member of the galactic system it represented the discoidal combination of several smaller clusters, the residual nuclei of which are still shown in the Orion, Perseus, and Scorpio-Centaurus groups of B stars.

[2] See the last three paragraphs of Section VII of this paper.

from the combination of spheroidal star clusters and has long been growing into its present enormous size at their expense. The evidence further suggests that the galactic system now moves as a whole through space, driving the spiral nebulae before it and absorbing and disintegrating isolated stellar groups. Apparently the suggested interpretation requires that two types of sidereal organization prevail generally throughout extra-galactic space: spiral nebulae, and stars of known types assembled for the most part into globular clusters;[1] and while the globular clusters now known are, at least potentially, members of the galactic system, the spirals are not members, rather being general inhabitants of extra-galactic space. The hypothesis demands that gravitation be the ruling power of stars and star clusters,[2] and that a repulsive force, radiation pressure or an equivalent, predominate in the resultant behavior of spiral nebulae.

[1] The apparent limitation of the size and mass of a globular cluster (the first of the five conditions) suggests, for example, the narrow range in masses of stars, for which the limiting factor, according to Eddington's theory, is the balance of attractive and dispersive forces. Judging by the galactic system, and perhaps by the local cluster and the Magellanic Clouds, the discoidal form permits a greater mass; and if, as seems likely, the Magellanic Clouds recently passed through the galactic system (cf. p. 12, n. 1, of the twelfth paper) a stellar discoid possibly shows much greater stability than is possessed by the compact spheroidal distribution.

[2] Jeans has considered mathematically the effect of the encounter of two clusters, showing the transformation from the globular to the oblate form (*Monthly Notices*, **76**, 552, 1916). His analysis may direct the way to an understanding of the beginning of a flattened stellar system of growing mass.

If such a growth be theoretically possible, we may suppose that at first the combination of separate clusters would proceed very slowly to the formation of composite systems of increasing mass, the galactic system that finally results representing an advanced stage in the survival of the most massive and stable. Once an enormous mass had been acquired, subsequent accretions would be numerous and almost inevitable if ordinary unit spheroidal clusters were encountered rather than growing discoids of multiple mass. The distinctly limited galactic star clouds, some of which are not mid-galactic, and the rapidly receding Magellanic Clouds as well, might represent partially assimilated and controlled systems of greater mass. Undoubtedly some globular clusters would merely describe orbits around the growing Galaxy and outside the limits of the system would cross the galactic plane, but our observations do not show them; it may be the true "orbital" clusters are constrained to keep far from the galactic system, all near approaches ending in disorganization. (Compare the tentative hypothesis sketched in the seventh section of *Mt. Wilson Contr.* No. 147, 1918.)

Relative to spiral nebulae, two additional conditions that seem not to have been considered heretofore may be pointed out as significant in connection with the proposed hypothesis.

a) In the midst of a field of stars the effect of repulsive forces would be largely nullified by the symmetrical distribution of the sources of repulsion (assumed to be the stars); above or below a discoidal stellar field the action is of course wholly one-sided.

b) The extremely high velocities of recession indicate that, if the galactic system had remained stationary, most of the brighter spirals would have been among the stars in low galactic latitudes within recent cosmic times, for instance within the last twenty million years. That scarcely any spiral (on either side of the galactic plane) is approaching or is now among the stars in low galactic latitudes not only suggests that a repulsive force is predominant, but also indicates either (1) a movement of the entire Galaxy through space, or (2) recently accelerated motions of spirals, or (3) distances enormously greater than now seem at all probable.

The last supposition, though extending the time allotted, would not remove the difficulty of accounting for the absence of spirals from the equatorial segment at the present time. For example, four bright spirals in high galactic latitude are receding with a velocity in excess of 1/300 that of light. Suppose they are as much as two million light-years away—ten times the distance of the remotest globular cluster known. Then, with constant velocities throughout the past, they would have been at the galactic plane less than 600 million years ago—an interval of time so short in the life-history of a stellar system that in the case of the sun, for instance, it has not sufficed to show an appreciable change in radiation.[1]

It is more in keeping, however, with observations of apparent rotation, and with the luminosity of novae in spirals, to divide by one hundred the distance supposed above. Only a few million years are therefore involved in the problem, and it seems all the more necessary to accept, in view of the observed distribution and radial motion, either (1) that the Galaxy is moving or (2) that the velocities have been rapidly developed. The suggestion (2) that

[1] *Publications of the Astronomical Society of the Pacific*, **30**, 283, 1918.

the position of the Galaxy has remained stationary while the velocities of spirals have but recently become large does not commend itself for various reasons, including the consequent necessity of seeking the origin of spirals in the galactic regions where now they are not observed.

The remarkable progression of average velocity with distance from the "galactic apex," discussed in Section VII of this paper, is also simply explained by assuming that the galactic system moves in the direction and with the velocity required by these phenomena of distribution and motion. We conclude, therefore, that the most plausible interpretation of the present arrangement and motions of the brighter spirals implies repulsion by the moving discoidal galactic system;[1] but we keep in mind the alternative hypotheses.

IX. NOTE RELATIVE TO THE MORE DISTANT B STARS OF THE LOCAL SYSTEM

The stars of spectral type B, because of their relatively small dispersion in absolute magnitude and their tendency to clustering and to community motion, are particularly valuable in studying the form and position of the local star cluster whose existence and general properties have been discussed provisionally in the last part of

[1] Is it possible that the spirals represent the failure to form stars from the original condensing nebulosity through the presence of too much material? According to Eddington's theory of the structure of a giant star (*Monthly Notices*, **77**, 16, 596, 1917), the pressure of radiation nearly counterbalances gravitation if the condensing gaseous mass exceeds some 10^{35} grams. A mass 100 times that of the sun would incompletely condense, possibly with the result of a diffuse pseudo-stellar nucleus, grading off into the extensive, low-density envelopes of gas that are unable to fall into the center because of the balance of radiation. Such a body would not readily disintegrate (since the ratio of radiation pressure to gravitation cannot exceed unity), but would present to external repulsive forces a surface relatively large for the mass involved, and composed, it may be, of particles of molecular dimensions peculiarly susceptible to the pressure of such radiation as is emitted by the stars. The existence of powerful electrical fields in spiral nebulae would clarify the problem, and already Slipher has suggested (*Lowell Observatory Bulletin*, No. 80, 1917) that such may occur in Messier 77 and Messier 1. Spirals of greatest mass or density would be least repelled by the galactic system, and if their individual velocities directed them toward a radiant source they might not be easily turned aside. In his study of the rôle of rotation in cosmogony Jeans has shown the probable development of spiral arms when the condensing body greatly exceeds the sun in mass (*Monthly Notices*, **77**, 186, 1917; *Scientia*, **24**, 270, 1918).

the twelfth paper of this series. Although the overwhelming majority of brighter B stars treated in *Mount Wilson Contributions*, No. 157 belong to the cluster, a few appear from the evidence of space positions to be field stars, in corroboration of Kapteyn's result that occasionally B stars are found in the second stream.[1]

It has frequently been assumed, mainly on the basis of an extrapolation, that all B-type stars are included in the earlier catalogues of spectra—that fainter than the seventh or eighth visual magnitude no stars of that class of spectrum would be found. Such a condition would place a sharp limit to the extent of the local cluster, assuming it to be outlined by stars of type B. Extremely faint and distant blue stars, however, have been found in the galac-

[1] The complete identification of the local cluster with Stream I involves some difficulties that were not sufficiently appreciated when the earlier paper was written. The two most important obstacles in the way of the suggested interpretation, which made star-streaming simply the result of uniform motion of the local cluster through the galactic field, appear to be the high relative velocity of the two streams and the probably different stream velocities for different spectral types. There seems to be no doubt of the existence of a definitely organized local cluster, and it certainly contains practically all of the brighter B stars. Kapteyn's studies also leave little doubt that the B stars as a whole, and therefore this cluster, move toward an apex approximately the same as the apex of the first stream as determined from stars of other types. The moving cluster must give rise to a true stream-motion. It happens that the position of the sun with respect to the local cluster's center (the direction of which is shown by F-G-K stars as well as by the B's) is such that we may have, in addition to and hardly distinguishable in direction from the true stream-motion, a pseudo-stream-motion due to internal circulation (according to Strömberg's suggestion, for instance).

These two possible sources of the observed preferential drift of stars were recognized from the first, but in the provisional discussion the internal motion was considered relatively of minor importance. Now it appears that the two difficulties mentioned above, and other minor ones, may be avoided if necessary by inverting the relative importance of the two sources of stream-motion. The cluster accordingly contains not only all the stars of Stream I but a considerable part of those of Stream II, and the cluster's direction and speed of motion are best measured by the mean drift of the early B stars. The average internal velocity may of course vary with spectral type without disrupting the cluster. Professor Eddington suggests in a letter that the field should probably be identified with Halm's O-stream, which contains a great many second-type stars formerly assigned to the first and second streams.

The modified view sacrifices little of the simplicity of the former statement and at the same time uses in a rational manner the large local cluster, whose existence must play an important part in the problem of star-streaming. Except in this matter of motion and content, none of the earlier conclusions referring to the local cluster appears to require appreciable modification.

tic clouds far beyond the limits of the local cluster, and the data given below show that Class B is also represented in considerable numbers among stars from the seventh to the tenth apparent magnitude.

The first volume of the *Henry Draper Catalogue*,[1] which covers the first four hours of right ascension, contains nearly 26,000 stars, of which 832 belong to Class B. It contains 600 B stars fainter than the seventh magnitude, corresponding to about 3 per cent of the total for all types. For the naked-eye stars over the whole sky the percentage is of course much larger, and, because of the galactic concentration of B stars, it will likely be larger also for the faint stars in those instalments of the new catalogue that include larger portions of the Milky Way. Practically all the B stars of this first volume are in Cassiopeia and Perseus.

Stars of the subdivisions B8 and B9 are generally discussed in connection with A-type stars. In the present note we shall only call attention to the following tabulation of the total number of such stars brighter than successive half-magnitude limits:

Visual magnitude	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5
Total No. stars...	7	17	32	56	95	156	235	317	430	537:	591::
Ratio.....		4.6	3.3	3.0	2.8	2.5	2.0	1.8	1.7:	1.4:	

The ratio N_{m+1}/N_m in the last line shows the decrease of density with distance.

The numbers of stars in the first five divisions of Class B are shown for different intervals of visual magnitude in Table VII. The limit to which the catalogue is complete is not specifically stated, but is probably in the neighborhood of visual magnitude 8.5. The 38 B stars with undetermined subdivision are more likely to belong in this table than with the stars of types B8 and B9.

Evidence discussed below indicates that the mean absolute magnitudes may be accepted as the same for all these faint stars as for the brighter B's, whose motions and parallaxes have led to the evaluation of their average intrinsic luminosities. Table VII indicates, therefore, that B stars are found continuously to a distance of possibly 1000 parsecs in the direction of Perseus and Cassiopeia.

[1] *Harvard Annals*, **91**, 1918.

TABLE VII

NUMBERS OF B-TYPE STARS IN *Harvard Annals*, 91

Spectral Type	Visual Magnitude							Total
	<7.0	7.0–7.5	7.5–8.0	8.0–8.5	8.5–9.0	9.0–9.5	>9.5	
B0	6	3	2	2	3	2	0	18
B1	4	2	1	0	1	0	0	8
B2	7	1	5	5	7	0	0	25
B3	26	11	7	5	7	1	2	59
B5	35	9	5	11	4	2	3	69
B0–B5	78	26	20	23	22	5	5	179
Undefined B	3	0	4	12	8	10	4	

Table VIII affords evidence that the 101 stars fainter than the seventh magnitude and of spectral types B0–B5 are possibly in large part members of the local cluster. The galactic latitudes, derived graphically from charts prepared by Kapteyn, are tabulated in order of decreasing brightness, the horizontal lines in each column marking the magnitude intervals of Table VII. All stars in the table are north of the celestial equator except the eight with galactic latitudes in excess of 41°.

With respect to the galactic plane, the descending node of the central plane of the local cluster is in galactic longitude 70°, approximately, as is clearly shown by Fig. 4 of *Mount Wilson Contributions*, No. 157. The galactic longitudes of the stars involved in Table VIII are almost exclusively between 85° and 130°, and, therefore, if they belong to the cluster rather than to the general galactic field, their latitudes should be predominantly negative. This is seen to be the case, and in fact we are led to believe that most stars of types B0, B1, B2, brighter than the tenth magnitude, may be members of the local system.

The area of the sky between right ascension 0^h and 4^h is mainly in the southern galactic hemisphere. Hence, for a fair comparison of the number of stars in positive and negative galactic latitudes, we should consider only a narrow belt along the galactic circle—the region within $\beta \leqq 5°$, for instance. We have then the following indication that the distribution of the early B-type stars of Table VIII may have little or nothing to do with the galactic

TABLE VIII

GALACTIC LATITUDES OF B-TYPE STARS FAINTER THAN MAGNITUDE 7

Bo	B1	B2	B3		B5	
+8°	−4°	+ 2°	− 3°	0°	− 8°	− 6°
+8	−3	− 3	− 4	+ 4	+ 2	+ 7
+1	−4	−12	+10	− 2	− 5	− 4
+6	−4	+ 1	−21	− 2	− 5	− 4
−2		− 5	−50	− 2	−12	− 6
−4		− 6	− 3	− 3	−23	+ 2
−4		− 2	− 2	− 5	−82	+ 4
−2		− 3	− 2	−42	− 3	+ 5
−4		−12	− 1	− 5	+ 1	− 5
−3		− 5	− 9	− 2	−12	−64
−4		− 5	−15	− 4	−16	− 6
−2		− 4	+ 2	− 5	+ 6	− 4
		− 3	+ 2	− 4	− 4	−63
		−19	+ 6	− 4	− 5	− 5
		− 4	− 3	− 6	−46	−54
		− 4	−54		− 7	− 6
		− 5	−16		+ 1	
		− 5	+ 2		+ 2	

plane—rather, these stars appear to be condensed to a circle that in this region of the sky is three or four degrees south of the galactic circle:

Spectrum		Bo	B1	B2	B3	B5	Bo–B5
Number of stars	β negative	8	4	12	18	10	52
	β positive	1	0	2	4	7	14

The maximum frequency of the galactic latitudes for the 101 faint stars of types Bo–B5 is about −4° according to the following tabulation, which is based on Table VIII:

β	≧+10°	+9°,+8°	+7°,+6°	+5°,+4°	+3°,+2°	+1, 0°	−1°,−2°
No. of stars	1	2	4	3	7	5	11

β	−3°,−4°	−5°,−6°	−7°,−8°	−9°,−10°	−11°,−12°	−13°,−14°	<−14°
No. of stars	28	19	2	1	4	0	14

It is easy to show that the dip of the equatorial circle of the cluster, due to the sun's position to the north of its central plane, is less than −1° for stars at the mean distance of those concerned above.

Allowing for the dip, and assuming that in the mean these B-type stars are about 25° from the descending node, we may make a rough determination of the angle between the central plane of

the local cluster and that of the Galaxy. The result is 8°, in fair agreement with the earlier value of 12°, but naturally of much lower weight. Accordingly, we may conclude, as previously assumed, that the existence and inclination of the local cluster do not depend upon the accidental positions of a few groups of brighter B stars.

The most significant feature of the preceding tabulation, however, is the high concentration to the central plane. More than half of all these stars fall into the interval of latitude $-2°$ to $-6°$, and three-fourths of the Bo, B1, B2 stars are within those limits. For B-type stars brighter than those considered here the dispersion is decidedly greater, both in this part of the sky and in general.

Three interesting conclusions may be drawn from the foregoing result: (1) The failure to find the fainter B stars heretofore may be due to the extremely narrow belts within which they are to be found—one belt, moreover, apparently standing well out of the lowest galactic latitudes for regions of small declination. (2) The local cluster is exceedingly flat, at least as far as the B-type stars are concerned. It may be more than five times as extended in its plane as at right angles. Table VIII shows how infrequent are the faint B stars in high galactic latitudes. (3) The greatly increased concentration for the fainter stars may be taken as proof that these objects, rather than peculiar B stars of abnormally low intrinsic brightness, are normal stars at a greater distance from the sun.

The completion of the *Henry Draper Catalogue* will afford a good basis for testing and extending the results outlined above.[1] With data for the whole sky we shall be able to define more accurately the position of the local cluster in the galactic system and perhaps determine its form completely.

MOUNT WILSON SOLAR OBSERVATORY
November 1918

[1] *Note added to proof:* Data derived from the second instalment of the *Henry Draper Catalogue* fully verify the existence of a secondary Galaxy as outlined by early B-type stars. The faintest B's, however, show a decided preference for the primary Galaxy, in contrast to the stars discussed above. Possibly a branching of the main Milky Way stream in Perseus and Cassiopeia, or the wide extent of the open clusters in Perseus, is largely responsible for the great preponderance of negative latitudes in the foregoing discussion; and accordingly these faint B's may be only in part members of the local cluster.

THE ECLIPSING BINARY RR VULPECULAE AND THE EVIDENCE OF DARKENING TOWARD THE LIMB

By MENTORE MAGGINI

A photometric light-curve of RR Vulpeculae has been published by me in an Appendix to *Pubblicazione di Arcetri*,[1] No. 34. This note will be devoted to the discussion of observations with a view to showing the evidence of absorption in the atmosphere and consequently of darkening toward the limb.

The light-elements derived from my observations were:

Primary minimum = J.D. 2420661.4749 + $5^d.051318$ E. (G.M.T.)

The individual 252 observations were collected into 42 normal points of equal weight. The first three columns of Table I contain the number of the normal, the phase referred to the epoch of primary minimum, and the mean magnitude.

From the observations we have:

Duration of light-change, $10^h50^m = 0^d.4514$
Duration of minimum light, $3^h20^m = 0^d.1389$
Stellar magnitude of constant light, $9^M.55$
Stellar magnitude of minimum, 10.87
No evidence of secondary minimum.

The range of variation at principal minimum, $1^M.32$, corresponds to a maximum loss of light of 0.7035 of the whole. The brighter star gives 0.7035 of the whole light of the system and, if isolated, would appear of magnitude $9^M.93$. If the eclipse were annular, the ratio of the stellar radii would be

$$k = \frac{\sqrt{1-\lambda_p}}{\lambda_s} = \sqrt{0.7035} = 0.845.$$

[1] Osservazioni di tre stelle variabili—Appendice di M. Maggini, *Pubbl. del R. Osserv. di Arcetri*, Fasc. 34. Firenze, 1916.

TABLE I

NORMAL MAGNITUDES OF RR VULPECULAE

No.	Phase	Mag.	O−C$_u$	O−C$_d$
1	−0^d.24169	9^M.55	0^M.00	0^M.00
2	.23600	9.55	.00	.00
3	.23068	9.55	+ .02	.00
4	.20847	9.60	.00	.00
5	.19236	9.70	− .02	.00
6	.18072	9.74	− .03	− .01
7	.15883	9.88	− .04	− .02
8	.15265	9.96	− .04	− .03
9	.13622	10.16	− .03	− .01
10	.12648	10.29	.00	.00
11	.11590	10.44	+ .03	+ .02
12	.10625	10.57	+ .04	+ .02
13	.09444	10.73	+ .07	+ .02
14	.08663	10.80	+ .07	+ .02
15	.07441	10.85	+ .05	+ .01
16	.07200	10.87	+ .04	+ .01
17	.05578	10.87	.00	.00
18	.03467	10.87	.00	.00
19	.02154	10.87	.00	.00
20	−0.00732	10.87	.00	.00
21	+0.01185	10.87	.00	.00
22	.02500	10.87	.00	.00
23	.03199	10.87	.00	.00
24	.04933	10.87	.00	.00
25	.06255	10.87	+ .01	.00
26	.07368	10.84	+ .04	.00
27	.08462	10.78	+ .07	+ .02
28	.10340	10.57	+ .05	+ .01
29	.11107	10.46	+ .03	+ .01
30	.12628	10.25	− .02	.00
31	.13680	10.14	− .03	− .01
32	.15135	9.94	− .04	.00
33	.15639	9.91	− .04	.00
34	.16078	9.86	− .04	.00
35	.17010	9.82	− .03	.00
36	.18617	9.70	− .02	.00
37	.19862	9.64	.00	+ .01
38	.20423	9.61	+ .02	.00
39	.21800	9.58	+ .02	.00
40	.22835	9.56	+ .01	.00
41	.23810	9.55	.00	.00
42	+0.25359	9.55	0.00	0.00

But if t is the semi-duration of the whole-minimum and τ the semi-duration of the constant-minimum light, we have

$$k \leqq \frac{t-\tau}{t+\tau} \qquad k \leqq 0.555$$

This shows that the eclipse is total. The light of the brighter star is therefore $L_b=0.704$, and that of the large fainter companion $L_f=0.296$. The loss of light intensity,

$$1-l=0.7035a,$$

corresponding to the various fractions a of the light of the brighter star, and the light remaining, expressed in stellar magnitude, are in the second and third columns of Table II.

A free-hand light-curve to represent the observed points is drawn, and the epochs t_1 and t_2 at which the magnitude computed is reached are read. Taking the period $P=5^d.051318$, the mean t of these times are transformed into orbital longitudes θ by the equation

$$\theta=\frac{2\pi}{P}t.$$

Using Russell's table[1] giving $\theta-\sin\theta$ we find $\sin\theta$ and $\sin^2\theta$. From the values of $\sin^2\theta$ we have:

$$A=\sin^2\theta_6=0.0286$$
$$B=\sin^2\theta_6-\sin^2\theta_9=0.0137$$

For every value of $\sin^2\theta$ we derive $\psi(\kappa, a)=\dfrac{\sin^2\theta-A}{B}$. The k_u (uniform) and k_d (darkened) are obtained from Russell's Tables II and IIx[2] giving the ψ-function.

We see that the values of k_u are discordant, but those for k_d are in good agreement. We have for their mean values:

$$k_u=0.262$$
$$k_d=0.473$$

New values of constants A and B are now deduced from this mean k. From Russell's Table II and IIx we obtain the ψ-functions for $k_u=0.262$, $k_d=0.473$, and a least-squares solution of the equations

$$\sin^2\theta=A_u+B_u\psi(0.262, a_1)$$

[1] *Astrophysical Journal*, **35**, 339, 1912, Table B.

[2] *Ibid.*, **35**, 335, 1912; **36**, 245, 1912.

TABLE II

a	$1-l$	Mag.	t_1	t_2	θ	$\sin^2\theta$	$\sin^2\theta-A$	$\psi(k_1a_1)$	k_u	k_d
0.0.....	0.0000	$9^M.550$	$-0^d.2307$	$+0^d.2381$	0.2916	0.0824	+0.0538	+03.927	0.488	0.488
.1.....	.0703	9.629	.2022	.2004	.2504	.0615	.0329	2.402	.416	.524
.2.....	.1407	9.715	.1853	.1829	.2290	.0515	.0229	1.671	.364	.501
.3.....	.2111	9.807	.1720	.1692	.2122	.0445	.0159	1.160	.350	.513
.4.....	.2814	9.909	.1590	.1570	.1965	.0380	.0094	0.687	.227	.415
.5.....	.3518	10.021	.1475	.1445	.1832	.0332	+ .0046	+0.337	.235	.450
.6.....	.4221	10.145	.1396	.1338	.1700	.0286	.0000	.....	.....	.....
.7.....	.4925	10.286	.1269	.1235	.1558	.0241	− .0045	−0.328	.273	.500
.8.....	.5628	10.448	.1150	.1118	.1411	.0198	.0088	0.642	.137	.400
.9.....	.6332	10.649	.1000	.0972	.1226	.0149	.0137	1.000	.....	.....
.95.....	.6683	10.748	.0917	.0880	.1117	.0124	.0162	1.183	.364	.533
.98.....	.6894	10.820	.0837	.0787	.1010	.0102	.0184	1.346	.220	.470
.99.....	.6965	10.845	.0775	.0720	.0929	.0086	.0200	1.460	.041	.420
1.00.....	0.7035	10.870	−0.0720	+0.0720	0.0833	0.0069	−0.0217	−1.584	0.030	0.457

and $$\sin^2\theta = A_d + B_d\psi(0.473, a_1^1)$$

furnish the new approximate values:

$A_u = 0.02931$ $\quad A_d = 0.02869$
$B_u = 0.01576$ $\quad B_d = 0.01417$

Two theoretical light-curves are computed with these constants. The results are given in Table III.

From the values t_1 and t_2 of Table II it appears that the epoch of mid-eclipse is $0^d.0016$ earlier than that of the light-elements.

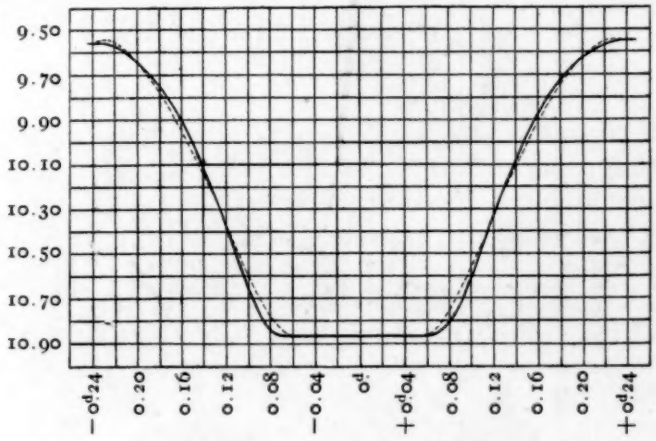

FIG. 1.—Light-curve of RR Vulpeculae
——— Observed and "darkened" light-curve
- - - - - "Uniform" light-curve

Plotting the magnitudes of Table II, in the first case against the epochs $-0^d.0016 \pm t_u$, in the second case against the epochs $-0^d.0016 \pm t_d$, we obtain two computed light-curves. They are shown in Fig. 1 and their residuals, $O-C_u$ and $O-C_d$, are in the last two columns of Table I.

The perfect agreement of the "darkened" solution with the observed curve is evident; the "uniform" solution is unsatisfactory.

Other elements were derived using the "darkened" solution. $\phi_1^1(k)$ and $\phi_2^1(k)$ were obtained from Russell's Table[1] II*ax*, and values of inclination i and radius of fainter companion r_1 calculated.

[1] *Astrophysical Journal*, **36**, 246, 1912.

TABLE III

Uniform and Darkened Light-Curve of RR Vulpeculae

Uniform $k_u=0.262$ $A_u=0.02931$ $B_u=0.01576$						Darkened $k_d=0.473$ $A_d=0.02869$ $B_d=0.01417$				
a	$\psi(k_u a)$	$B_u\psi(k_u a)$	$\sin^2\theta$	θ	t_u	$\psi(k_d a\sqrt{})$	$B_d\psi(k_d a\sqrt{})$	$\sin^2\theta$	θ	t_d
0.0.....	+2.972	+0.04685	0.0762	0.2795	$0^d.2247$	+3.870	+0.05483	0.0835	0.2931	$0^d.2356$
.1.....	2.056	.03240	.0617	.2509	.2017	2.288	.03242	.0611	.2497	.2007
.2.....	1.532	.02415	.0535	.2332	.1875	1.637	.02319	.0519	.2298	.1847
.3.....	1.086	.01712	.0464	.2172	.1746	1.127	.01597	.0447	.2129	.1712
.4.....	0.704	.01110	.0404	.2023	.1626	0.708	.01003	.0387	.1979	.1591
.5.....	+0.342	+ .00539	.0347	.1873	.1506	+0.341	+ .00483	.0335	.1841	.1480
.6.....	0.000	.00000	.0293	.1720	.1382	0.000	.00000	.0287	.1702	.1368
.7.....	−0.327	− .00515	.0242	.1562	.1256	−0.325	− .00461	.0241	.1558	.1252
.8.....	0.654	.01031	.0190	.1383	.1112	0.651	.00922	.0195	.1400	.1125
.9.....	1.000	.01576	.0135	.1167	.0938	1.000	.01417	.0145	.1208	.0971
.95.....	1.195	.01883	.0105	.1026	.0825	1.192	.01689	.0118	.1088	.0875
.98.....	1.337	.02107	.0082	.0909	.0731	1.345	.01906	.0096	.0983	.0790
.99.....	1.401	.02208	.0072	.0851	.0684	1.422	.02015	.0085	.0925	.0744
1.00.....	−1.500	−0.02364	.0057	.0754	.0606	−1.575	−0.02232	.0064	.0799	.0642

Relative surface brightness $\frac{J_b}{J_f}$ and "equal-mass" densities $\bar{\rho}_b$, $\bar{\rho}_f$ are derived from the formulae:

$$\frac{J_b}{J_f}=\frac{L_b r_f^2}{L_f r_b^2}$$

$$\bar{\rho}_b=(5.29P^{\frac{1}{2}}r_b)^{-3} \qquad \bar{\rho}_f=(5.29P^{\frac{1}{2}}r_f)^{-3}$$

The quantities in parentheses are the radii $\bar{r}_b$,$\bar{r}_f$ of the stars relative to the sun.

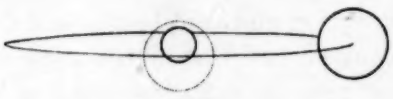

FIG. 2

Finally the hypothetical parallax was computed by Shapley's method.[1] Since the spectrum of RR Vulpeculae is unknown, it was estimated to be of type F from the consideration of hypothetical radius $\bar{r}_b > 1.2$. Definitive "darkened" elements of the systems are as follows.

RR VULPECULAE

Radius of bright component	r_b	0.096
Radius of faint component	r_f	0.202
Inclination of plane of orbit	i	86°0′
Least apparent distance of centers	$\cos i$	0.070
Light of bright component	L_b	0.704
Light of faint component	L_f	0.296
Relative surface brightness	J_b/J_f	10.53
"Equal-mass" density of bright component	$\bar{\rho}_b$	0.304
"Equal-mass" density of faint component	$\bar{\rho}_f$	0.032
Radius of bright component	$\bar{r}_b$	1.488
Radius of faint component	$\bar{r}_f$	3.146
Hypothetical parallax	$1000\,\pi''$	3.16

Fig. 2 shows the system of RR Vulpeculae as seen from the earth.

FLORENCE, ITALY
December 1918

[1] "A Study of the Orbits of Eclipsing Binaries," *Contributions from the Princeton University Observatory*, No. 3, pp. 117–18.

Minor Contributions and Notes

THE RESIDUAL RADIAL VELOCITIES OF THE CEPHEID VARIABLES AND THEIR BEARING ON A PULSATION THEORY

In an article in this number on the cause of Cepheid variation[1] I called attention to the fact that if this type of variation were due to pulsation effects, the radial velocities of the systems upon the hypothesis of orbital motion should show a considerable negative excess, which was not the case in the data then available. This seems to be a crucial test of such a theory and to merit a more careful examination of the data, which, although limited, is now somewhat increased, and sufficient to give confidence in the result.

For such an examination 21 stars are now available. The essential data are given in the table on page 149.

The solar motion was eliminated on the basis of 20 km per sec. toward $\alpha = 270^\circ$ and $\delta = +30^\circ$. V' is the corrected velocity.

The mean velocity of the 21 Cepheid stars given in the table, having regard to sign, is -4.4 km for all, or -2.0 km omitting RR Lyrae, whose velocity is 50 km. Regardless of sign it is 12.2 km, or 10.4 omitting RR Lyrae. The mean radial velocity of nearly two hundred stars of types F and G, with small proper motions, is 10.3 km. In good agreement with this are the values for the Cepheids upon the assumption of orbital motion. The average radial velocity of these Cepheids upon a pulsation theory would be 16.6 km, or 15.6 km omitting RR Lyrae.

The tendency appears now to be to admit the variations of radial velocity observed in the Cepheids as true Doppler-Fizeau effects, that the pulsations are radial translations of an outer shell. If such is the case and the mean algebraic velocity of the group of

[1] *Astrophysical Journal*, **49**, 81, 1919.

stars is actually zero, then the mean velocity of the group derived in the usual way from the *mean* of the observed variable radial motions should be -16 km (*minus* K) instead of -4.4, or -2.0 km.

For it is clear that upon the assumption of pulsation the total observed variation of radial velocity is nothing more nor less than

TABLE I

CEPHEID VARIABLES

Star	1900.0 α	1900.0 δ	K	Obs. V	V'
			km	km	km
Polaris	$1^h\ 22^m$	$+88°\ 46'$	3	−15	− 5
SU Cassiopeiae	2 43	+68 27	11	− 7	− 2
SZ Tauri	4 31	+18 20	11	− 3	−16
RT Aurigae	6 22	+30 34	18	+21	+12
ζ Geminorum	6 58	+20 43	13	+ 7	− 5
S Muscae	12 7	−69 36	18	+ 3	− 5
R Triang. Austr.	15 10	−66 8	18	−20	−24
S Triang. Austr.	15 52	−63 30	15	+ 1	− 2
S Normae	16 10	−57 39	14	−10	−10
RV Scorpii	16 51	−33 27	15	−28	−20
X Sagittarii	17 41	−27 48	15	−14	− 3
Y Ophiuchi	17 47	− 6 7	8	− 5	+11
W Sagittarii	17 58	−29 35	20	−29	−19
Y Sagittarii	18 15	−18 54	19	+ 4	+17
RR Lyrae	19 22	+42 6	22	−69	−50
SU Cygni	19 40	+29 1	25±	−33	−15
η Aquilae	19 47	+ 0 45	21	−14	+ 2
S Sagittae	19 51	+16 22	19±	−12	+ 5
T Vulpeculae	20 47	+27 52	18	− 1	+15
β Cephei	21 27	+70 7	17	− 5	+ 8
δ Cephei	22 25	+57 54	20	0	+13
Mean			16.1		− 4.4
Mean omitting RR Lyrae					− 2.0

the amplitude of the pulsation, that the true radial motion of the star is not the *mean* of the curve, but entirely on the side of the greatest positive motion. The motion would be first all approach and then all recession, and by taking the mean of the velocity-curve we obtain, not the velocity of the star, but a quantity equal to *minus* K. In the case of the 21 Cepheids under discussion the mean V' is -4.4 km, or -2.0 km, which is much closer to the required zero for orbital motion than to the -16 km required by pulsations.

The magnitude of the difference between the residual velocity of the group, on the two assumptions, adds to the strength of the test, and pronounces definitely in favor of duplicity and against pulsation. This conclusion is further confirmed by the individual velocities, 8 of which are positive; on the assumption of pulsations practically all should be negative. In order to harmonize these results with pulsations it would be necessary to assume an average outward motion of from 12 to 14 km for these Cepheids, which seems highly improbable. Neither does a change of solar velocity from 20 km to 16 km, which is that derived from the stars of types F and G, greatly change the result. The use of 16 km gives a residual for the 21 stars of -5.7 km, or -3.2 km omitting RR Lyrae.

The objection to duplicity of the Cepheid stars appears to rest chiefly upon the small dimensions of their orbits in connection with their supposed giant size. The orbital dimensions are free from uncertainties of distance. The sizes of the Cepheids appear to rest finally upon distances derived from their small proper motions. I have already referred[1] to recent investigations at Mount Wilson and this Observatory which throw doubt on proper motion as a measure of distance, particularly of the stars with such small proper motions as the Cepheids. The work here is farther advanced than when the foregoing article was written, and the explanation of parallax for the discordances found is now strengthened. If the present suspicions as to parallax are confirmed the stars with small proper motions are in general much closer than has been assumed. In such a case the supposed giant sizes of the Cepheids are likely to be reduced to such an extent that the small orbits found for them would present no serious difficulty.

C. D. PERRINE

OBSERVATORIO NACIONAL ARGENTINO, CÓRDOBA
March 5, 1919

[1] *Astrophysical Journal*, **48**, 296, 1918.

REVIEW OF RECENT WORK ON THE SERIES SPECTRA OF HELIUM AND OF HYDROGEN

At the time of the publication of Kayser's *Handbuch der Spektroscopie* it seemed that we knew everything about the spectrum of helium. Since then, however, several articles have appeared which throw much new light on the subject, and make it seem that there may even yet be more to learn. As these have been rather widely scattered, and have received less attention than they deserve, it may be worth while to review them briefly at this time.

Runge and Paschen[1] showed that the spectrum of He consists of two systems of series, a system of pairs, sometimes referred to as the He I spectrum, and a system of single lines, the He II spectrum, which was at one time referred to a pseudo-element parhelium—a hypothesis which was soon abandoned, though the term has survived. Each series system consists of four types of series, one principal, two subordinate (diffuse and sharp), and a fourth type which we may, following Hicks, refer to as the fundamental series. German writers frequently call it the "Bergmann" type, though the first series of this sort was published by Fowler, and the true nature of the type first discovered by Runge.

Fowler[2] and Bohr[3] have shown that there is also an "enhanced" or "spark" series system, which includes two series formerly supposed to belong to H. This system is emitted by an atom which has been more violently bombarded than is usual in ordinary sources. Stark would have us believe that it is emitted by a helium atom which has been deprived of two electrons. Such an atom would presumably be an alpha particle, and it is difficult to imagine in the light of present evidence how such a particle could emit any light-waves.[4] On various grounds it seems preferable to assume, as Bohr does, that the ordinary He spectrum is given by an atom which has, after having been deprived of one electron, just regained it; while the enhanced system comes from those atoms which,

[1] *Astrophysical Journal*, **3**, 4, 1896.

[2] *Monthly Notices*, **73**, 62, 1912. [3] *Nature*, **92**, 231, 1913.

[4] It is here assumed that, even if we follow Rutherford's recent suggestion and think of the He nucleus as containing electrons, these could still furnish no ordinary light, as they would be too rapid in their motions.

having lost both electrons, have just regained one. Possibly Stark's own observations could be reconciled with such a view. Stark also suggests that the single-line and pair systems may be due each to the vibrations of one of the two electrons in the He atom; but it might be better to suppose that the atom is symmetrically constructed, and that the two types of vibration are executed in different directions with respect to the axis of symmetry.

The enhanced system consists of lines at first thought to be pairs, but Paschen[1] has shown in a very important article on "Bohr's Helium Lines" that they are complex, and there seems to be little hope that this complexity can be explained away by the assumption that it is due to Stark effect, or to any other disturbing cause. Sommerfeld's modification of Bohr's theory, on the assumption that the electronic orbits are elliptical rather than circular, furnishes a quantitative explanation of the complexities here discovered. This is one more strong argument in favor of the rotating atom, though one cannot lightly abandon the hope that the conception of an atom such as that of Lewis and of Langmuir,[2] which explains the chemical behavior of matter so much better, can be so enlarged as to fit the requirements of spectroscopy as well.

According to modern theories, the He atom, when deprived of one electron, should bear a strong resemblance to that of H. The enhanced series system of He bears out this idea perfectly, for it consists, as far as we now know it, of two series, which are related to one another in exactly the same manner as are the series in H. Further, they may be calculated by a formula of the Balmer type, in which $4N$ is used instead of N, though the value of N must be altered slightly from that necessary for H. One of these series is an exact copy of the other, merely shifted to another part of the spectrum. Stark has just shown that the electrical resolution of the main He enhanced series is identical, line for line, with that of the Balmer series of H. The main series of He, beginning with the famous 4686 line, lies largely in the ultra-violet, and includes what was formerly supposed to be the principal series of H. The other series, lying in the visible spectrum, includes, as every other line, the

[1] *Annalen der Physik*, **50**, 901, 1916.

[2] *Jour. Amer. Chem. Soc.*, **41**, 868, 1919.

Pickering series, also formerly supposed to belong to H. The alternate lines lie close to those of the Balmer series of H, but were successfully measured by Evans.[1] Other shifted series of this sort may also exist, and their positions can readily be predicted.

In a recent number of the *Annalen*[2] there are four articles by Stark and his pupils on the He spectrum, in which series of lines, largely new, are given, generated in that part of the source where the field is strongest. An earlier discovery of the same sort was that by Koch[3] of a "third subordinate series of He." This designation is misleading, but has been handed down from the time before the combination principle of Ritz came into general use. This series is a combination series of the principal rather than the subordinate type. Stark's series are also combination series, but he has applied new names to them, at variance with current custom, which will add confusion to the already somewhat mixed nomenclature of this subject. He calls all series principal series which run to the same limit as the principal series does. He thus gets "diffuse principal," "sharp principal," and even "near-sharp (fastscharfe) principal" series. Heretofore the nature of a series has always been supposed to be determined, not by the position of its limit, but by the spacing of its lines, their physical aspect, etc. That this is the proper procedure Stark himself has just shown, for he proves that the Stark effect for his "diffuse principal" series is identical with that of the diffuse series from which (by combination) it is derived. In order to help in translating these misleading designations back into the ordinary nomenclature, both sets are shown in the table below. It is a remarkable fact that these new series of Koch and of Stark appear in the strongest electrical fields only, and yet they belong, not to the enhanced system, as this would indicate, but to the other systems, whose lines are produced by a quite moderate degree of excitation. The same remark applies to the (erroneously) so-called "third subordinate series" of Li.

Recently Fowler[4] has found a remarkable set of bands in the He spectrum, some of which follow a line-series formula. They are

[1] *Philosophical Magazine*, **29**, 284, 1915.

[2] *Annalen der Physik*, **56**, 569–617, 1918.

[3] *Ibid.*, **48**, 98, 1915.

[4] *Proc. Roy. Soc.*, **91**, 208, 1915.

TABLE I

The Helium Series System

System	Name	Series "Formula"	First Lines	Remarks
System of single lines or He II or Parhelium	Principal	$(1S)-(mP)$*	20581, 5016, 3965	Stark's "near-sharp principal" series
	Sharp or second subordinate	$(1P)-(mS)$	7282, 5048, 4438	
	Diffuse or first subordinate	$(1P)-(mD)$	6678, 4922, 4388	
	Fundamental	$(1D)-(mF)$	18694, 12793	Not mentioned in Stark's summary of He spectrum.
Combinations in this system		$(1P)-(mP)$	6635, 4911, 4384	Lines observed by Merton also
		$(1S)-(mS)$	(5380) (4053) (3651) 3468	Bracketed lines not yet observed; Stark's "sharp principal" series
		$(1S)-(mD)$	(5043) 3974, 3617	Stark's "diffuse principal" series
		$(2P)-(mD)$	19096	One line, found by Paschen.
System of pairs	Principal	$(1s)-(mp)$	10830, 3889, 3188	Stark's "near-sharp principal" series
	Sharp	$(1p)-(ms)$	7066, 4713, 4121	
	Diffuse	$(1p)-(md)$	5876, 4472, 4026	
	Fundamental	$(1d)-(mf)$	18684, 12785	Found by Paschen
Combinations in this system		$(1p)-(mp)$	6060, 4518, 4046	Koch's "third subordinate" series Stark's "near-sharp subordinate" series
		$(1s)-(ms)$	(4277, (3285) 2986	Stark's "sharp principal" series
		$(1s)-(md)$	3809†, 3166, 2936	Stark's "diffuse principal" series
		$(2p)-(md)$	17008	Found by Paschen.
Enhanced system	Fowler's main series	$4N\left(\frac{1}{3^2}-\frac{1}{m^2}\right)$	4686, 3203, 2733	Includes series formerly called principal series of H
	Pickering	$4N\left(\frac{1}{4^2}-\frac{1}{m^2}\right)$	6560, 5411, 4859	Includes series formerly called the Pickering series of H.

* For an explanation of these symbols see *Astrophysical Journal*, 41, 324, 1915.

† The first line of this combination series was observed by Paschen and is given in the tables of Dunz.

not included in Table I, as their relationship to the other series is unknown.

It should be added that Merton[1] in the course of experiments on the Stark effect in He recognized at least one of the combination series which Stark (a few days later) published, but apparently did not definitely identify the combination which gives it.

For the sake of completeness the spectrum of the H atom, as far as it is now known, is appended. The second, or compound, spectrum of H is not here given, as it is now generally regarded as due to the H molecule, and it is not known to contain any line series.

TABLE II

Designation	Formula	First Lines
Ritz series.....	$N\left(\frac{1}{2^2}-\frac{1}{m^2}\right)$	1216, 1026
Balmer series.....	$N\left(\frac{1}{3^2}-\frac{1}{m^2}\right)$	6563, 4861, 4340
Paschen series	$N\left(\frac{1}{4^2}-\frac{1}{m^2}\right)$	18751, 12817

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August 1919

THE ABSORPTION SPECTRUM OF CARBON DISULPHIDE VAPOR

The absorption spectrum of carbon disulphide vapor seems to have been investigated twice, viz., by Liveing and Dewar,[2] who found an absorption band between λλ 3400 and 3000 for very dense vapor and between λλ 3360 and 3020 for a less vapor-density, and by J. Pauer,[3] who resolved the band between λλ 3300

[1] *Proc. Roy. Soc.*, **95**, 30, 1918.

[2] *Ibid.*, **35**, 714, 1883.

[3] *Annalen der Physik*, **61**, 363, 1897.

and 3000 into lines and measured the wave-lengths of fourteen of these lines.

Pauer used as his source of illumination the light from a cadmium arc, which is not continuous in this spectral region.

In the present investigation the source of illumination was a spark between aluminium electrodes under water. The apparatus used was modified from a description given by H. E. Howe.[1] The spark was produced in a vessel of hard rubber and the light was allowed to emerge through a window of two quartz lenses, which gave a nearly parallel beam of light to the slit of the Féry quartz spectroscope. This spectroscope gave a spectrum about eighteen centimeters long in the ultra-violet.

The vapor was contained in a glass cylinder 9 cm long with quartz windows at the ends. The carbon disulphide was admitted through a side tube and allowed to evaporate in the vessel.

Photographs were made with some liquid in the bottom of the vessel at room temperature, and others after the liquid had all evaporated and much of the vapor had been driven out by warming the absorption vessel. The spectrum from a copper arc was used as a comparison spectrum from which to measure the wave-lengths of the lines. These measurements were made on a Hilger plate-measuring engine, and are probably accurate to one or two units in the fourth place.

With the saturated vapor-density at room temperature the absorption was complete throughout the greater part of the band, and it was only by using a very low vapor-density that the single lines could be made out in this part of the spectrum. In the case of even very dilute vapor the absorption was complete beyond λ 2230.

On account of the feeble illumination, long exposures were necessary to give a continuous spectrum to the limits set by the quartz apparatus. The exposures were accordingly about seven hours. The following lines were determined in the different photographs.

[1] *Physical Review*, **8**, 574, 1916.

PAULY	PAUER	PAULY	PAUER	PAULY	PAUER
2919		3074		3259	
2931		3081		3264	
2939		3084		3268	3270
2944		3090	3091	3275	
2949		3093		3284	
2956		3103		3291	
2960		3110	3110	3302	3300
2967		3119		3306	
2973		3133	3133	3309	
2975		3135		3322	
2982	2982	3138		3326	
2996		3142	3140	3331	
3000		3150	3147	3348	
3010	3010	3154		3356	
3013		3156		3368	
3016		3158		3383	
3018		3165	3162	3388	
3020		3174	3175	3391	
3024	3024	3183	3182	3406	
3028	3027	3190		3421	
3034		3206	3202	3438	
3039		3215	3210	3453	
3043	3042		3220	3471	
3046	3046	3227	3227	3499	
3054		3235		3508	
3057		3246	3245	3524	
3065	3065	3252	3252	3547	
3067				3554	
3070	3070			3577	
				3589	

IRENE M. PAULY

STANFORD UNIVERSITY
June 18, 1919

We are pained to learn from a cable dispatch to an Italian paper published in New York that death has overtaken our esteemed collaborator

PROFESSOR ANNIBALE RICCÒ
Director of the Observatory of Catania

We shall publish later a sketch of the life and fruitful scientific activity of this distinguished astrophysicist.

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